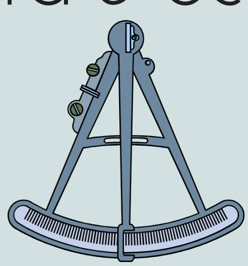
Southeast Math

Grade Levels 7 – 12

Algebra & Geometry



Goldbelt Heritage Foundation

"Drink from the Vessel of Traditional Knowledge."

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Southeast Math

7-12 Algebra/Geometry

Credits

Author: Paul Berg
Math Advisor: Ward F. Ward
Photo Credits: Virginia Berg
Editor: Tiffany La Rue

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Algebra Lesson 1

Tlingit Phrase: Yáadu X̲áat!

English Translation: The fish are here!

Lesson 1 - Linear Equations

The earth revolves around the sun in an annual cycle, creating the seasons. The Northern Hemisphere shifts through the annual cycle receiving the sun's energy in varying amounts. Alaska receives the most solar energy when the earth's axis is tilted toward the sun. This takes place during the months of May, June, July and August.

Summer in Alaska, when the days are long and the ground warms from greater periods of exposure to the sun's heat, is when all living things become more active. Seasons unfold and life cycles in Alaska become intertwined. Salmon come home, driven by an undeniable desire to return to the very stream in which they were born. They return by the millions. Bears and eagles congregate to the streams and we humans break out our nets and fishing rods in pursuit of this precious food source.

All life--plants, and animals--are connected. The world is such a complicated place that it is beyond our capability to precisely describe all the connections or every action/reaction. There are just too many **variables**. This is where mathematics can help us. If we don't know what a particular quantity is, we can assign the unknown quantity a name, such as X. Once we assign a variable (X) to represent an unknown quantity, then we can fit the unknown in to its rightful place in an equation.

In the following example, we do not know the weight of each and every fish that John caught, but we can call the weight of a single fish X and then place X into an equation which describes what we do know about the fish. Let's examine several situations where we have an unknown number. We will call this unknown number **X**.

SOLVING LINEAR EQUATIONS

Yesterday John caught a fish. He stepped onto a scale while holding his fish and the scale read 192 pounds. If we know that John's weight is 175 pounds, how much did his fish weigh?

____

(answer)

You can easily answer this question. Intuitively you might think that if 192 lbs. is the total weight, and 175 lbs. is John's weight, then subtracting his weight from the total will give the weight of the fish: 192 lbs. – 175 lbs. = 17 lbs. The weight of the fish is 17 pounds!

Let's look at the problem another way. The total weight is John's weight plus the weight of the fish. We don't know the weight of the fish, so let's call it X. This situation can be written as an equation:

The fish's weight plus John's weight is the total weight.

$$X + 175 \text{ lbs.} = 192 \text{ lbs.}$$

We take away 175 lbs. from each side to solve for X = 17 lbs.

$$X + 175 \text{ lbs.} - 175 \text{ lbs.} = 192 \text{ lbs.} - 175 \text{ lbs.}$$
$$X = 17 \text{ lbs.}$$

The idea is to get X by itself. The kicker is that whatever we do to accomplish this <u>must be done on both sides of the equal sign</u>.

Activity 1

This time John caught two fish. The second fish is five pounds heavier than the first. When John steps on the scale holding his two fish, the scale reads 197 pounds. Can you find the weight of each fish? Give it a try:

____
(answer)

How did you do? Maybe your intuition carried you through. Maybe you used algebra. Maybe the two are not so different.

Let's try an algebraic approach to solving this problem.

We don't know the weight of either fish, so we start by assigning a variable to the weight of one of the fish:

Let X = weight of the first fish
Then we know X + 5 = weight of the second fish

Here's what we know:

The weight of the first fish plus the weight of the second fish plus John's weight equals the total weight.

$$X + (X + 5) + 175 = 197$$

Let's solve this equation together. Combine the terms on the left side of the equation. Our equation now looks like this:

$$2X + 180 = 197$$

Then subtract 180 from both sides of the equation.

$$2X + 180 - 180 = 197 - 180$$
$$2X = 17$$

Now, divide both sides by 2. X = 8.5 lbs. This is the weight of John's first fish.

X + 5 = 13.5 lbs. This is the weight of John's second fish.

Activity 2

James and Tim have set up camp near a small stream on Admiralty Island. They have stowed their gear carefully, with no food or fishing tackle near the camp. Coho are running up stream at a rate of 8 fish per hour. Tim estimates that the Coho vary in weight from 8 to 12 pounds. Assuming that the average Coho is 10 pounds and that the run continues at the same hourly rate, what will be the total biomass *(that is, the weight of all the returning Coho)* added to the headwaters of the stream during a ten day period? *(Hint--let X be the weight of the biomass.)*

Total weight (*biomass*) of returning Coho in ten days is ___________________

The whole ecosystem, including bears, eagles, and even hungry halibut downstream feed directly off the dead Cohos. Even a modest salmon run in a small stream adds a large amount of nutrients to the forest ecosystem. All life in the rainforest benefits from this yearly infusion of nutrients.

Did you know that the salmon eggs will not hatch if the water in the stream becomes too warm? Overhanging branches from trees are essential to shade the stream and maintain the cool environment needed by the eggs. Two weeks of uninterrupted sunshine in August may please the tourists, but the effect on salmon eggs can be disastrous.

Activity 3

The female Coho salmon digs a nest with her tail and deposits from 2,400 to 4,500 eggs. Assuming that half of the returning salmon are female, how many eggs will be deposited in the stream during that ten day period of time? (*Hint—use the data from Exercise 2. First determine the number of Cohos returning during the ten day period and divide by two. Assume that the average female deposits 3,450 eggs. Let X be the total number of eggs.*)

Number of eggs deposited in ten days: ____________________

Salmon Eggs

Activity 4

Several days after camping near the stream, James learned that two years previously an Alaska Department of Fish and Game biologist had conducted a study of this stream. According to the biologist, bears are particularly active near this stream during the ten day Coho run. Predation of the spawning Cohos from bears is estimated to be four percent of the run. In that same ten day period, assuming the run is steady, how may Cohos *(both male and female)* will be taken by bears? How many eggs will be lost to bears?

Number of Coho taken by bears: ______________________

Number of eggs lost to bears: ______________________

Algebra Lesson 1 Extension

Locate a nearby salmon stream. Ask parents, grandparents, and Elders about the history of the salmon runs in the stream. Have the runs changed over time?

Do an actual stream study. Count the number of each species of salmon passing a given point in a half hour period for a specific number of days--five to ten days. If you can, sample several fish and determine an average weight. Determine the duration of the salmon run. Based on the data which you have gathered, calculate the biomass of the salmon run and the number of salmon eggs deposited in the stream. If you have an active bear population, you can also factor predation into this calculation.

Sockeye Salmon Spawn

Algebra Lesson 2

Tlingit Phrase: Goosóo wé aas gutóode?

English Translation: Where is the forest?

Lesson 2 - The Amazing Life of Trees

Southeast Alaska is the home of the Tongass National Forest - the largest area of temperate rain forest on the planet. If you have ever hiked through a lush forest, you may have been struck by a sense of timelessness; a pervading calm that removes you from every day life. The forest is a world at peace, where change and growth happens over years, decades, and centuries. In our oldest forests in Southeast Alaska, trees may live for more than 800 years, gain a diameter of 12 feet, and grow to a height of 200 feet or more! Imagine the life of such a tree.

- During the tree's lifetime, the forest beds have been thick with vegetation, tightly fixing the ground with intermingling root systems.
- Local deer will have sought out the shelter of the tree from storms and fed on the undergrowth pro tected by its canopy.
- Returning salmon have found ideal spawning grounds in streams near this tree.
- The temperature of the stream is regulated by the shade of this great tree and its neighbors.
- The stream's current is broken by branches and fallen trees, making a perfect refuge for laying eggs.
- The salmon in streams near this ancient tree have flourished because of these conditions, thus adding to the health of oceans.
- Life gives life, as bear, eagles, and other birds feed on the salmon.

During this tree's existence, life abounded in the region. Local **Tlingit**, **Haida**, and **Tsimshian** tribes have enjoyed abundance. In the last 100 years, many of its neighboring trees have fallen to a logger's blade and a growing timber industry. Left to natural cycles, the forest weaves an intricate balance to sustain a complex ecosystem, allowing the diversified animal and plant life of Southeast to prosper. In a world where time is measured on a greater scale, day to day events become diluted in a much larger pool of time. Giants of our world, the old growth trees are elders to us all. With them, we can find solace. With them we can find the wisdom that comes with age.

In the last fifty years, much of the old growth forest of Southeast Alaska has been logged. Major portions of the Tongass Rainforest have experienced a reduction in old growth forest and the ability of the land to support life. As the large old growth trees are harvested, the rich ecosystem they support deteriorates. The land becomes less able to support deer, wolves, eagles, and bear.

HOW OLD ARE OUR TREES?

The best way to determine a tree's age is to count the annual rings. Each ring represents one year of growth. The lighter and wider part of each ring grows during spring and summer, the plentiful time of year, when growth is fast. The darker part of each ring is added during fall and winter, when nutrients are not as plentiful and growth is slower.

Each ring, from light to dark, represents the cycle of seasons for one full year. As you can see in the picture, each year's ring varies in width. The innermost rings are from when the tree was young and growing very fast. The outer rings are more recent when the tree has aged and its outward growth has slowed considerably.

The rings of a tree provide us with a snapshot into the past, a history of climate conditions over the life of the tree. Ring widths depend on many factors, like the species of the tree, how much competition the tree has for sunlight, regional climate, annual rainfall, etc. The weather conditions during a particular year are reflected in the width of tree ring. Each species of tree will have an average ring width. Foresters use this information to determine a tree's approximate age by measuring the tree's **diameter**.

Activity 1

The forested area off the end of Thane Road in Juneau has been logged. If you walk through this area, you would notice that most of the trees are similar in size. This is because they are all about the same age.

You are investigating the age of a stand of trees on Thane Road. In order to get an *average* age, you decide to measure the circumference of 10 trees. The **circumference** of a tree is the distance around it, and you decide this would be the easiest measurement to take. Your tape measure only measures in feet and tenths of a foot. These are the measurements you have taken:

Table 1

Tree #	1	2	3	4	5	6	7	8	9	10
Circumference (in feet)	$4\frac{8}{10}$	$5\frac{3}{10}$	$4\frac{2}{10}$	$5\frac{1}{10}$	$4\frac{3}{10}$	$5\frac{5}{10}$	$4\frac{7}{10}$	$5\frac{6}{10}$	$5\frac{2}{10}$	$4\frac{9}{10}$

Since 1 foot = 12 inches, we can change from feet to inches by multiplying by 12.
For tree #1 this gives us:

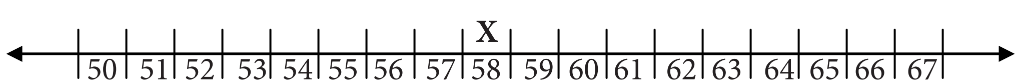
$$4\frac{8}{10}x12 = \frac{48}{10}\cdot\frac{12}{1} = \frac{48}{5}\cdot\frac{6}{1} = \frac{288}{5} = 57\frac{3}{5}in$$

For trees #2 — 10, change from feet to inches. Keep your work in fraction form and enter the answers in **table 2**.

Table 2

Tree #	1	2	3	4	5	6	7	8	9	10
Circumference (in inches)	$57\frac{3}{5}$									

One way to display the measurements in table 2 is to make a **line plot**. Round each of the measurements in table 2 to the nearest inch and put an X above the appropriate number on the line below. The measurement for tree #1 has been done for you.



Now that we have the Circumference, C, for each of our ten trees, we can use these measurements to find the Diameter, D, for each tree. Tree diameter is measured at chest height, or about 4.5 feet above ground level.

The Circumference of a circle is the product of the circles Diameter and pi (∏). Multiply the circle's Diameter by ∏ to determine the Circumference. Translate this last sentence into a formula involving C and D and write the formula on the line below.

____

Another way of stating the last formula is to say that the Diameter, D, of a circle is the ratio of its Circumference, C, and pi. This formula can be written as follows:

$$D = \frac{C}{\pi}$$

Since we now have the Circumference for each tree in **Table 2**, we can use this formula, and the fact that ∏ is approximately 3.14 (∏ ≈ 3.14), to calculate the Diameter of each of our ten trees.
Use the information from Table 2, and the formula $D = \frac{C}{3.14}$, to calculate the Diameter of each tree.

You should change each measurement to a decimal first, then use your calculator. Round each diameter to the nearest tenth of an inch.

For tree #1 this gives: $57\frac{3}{5} = 57.6 \text{ in.}$

The Diameter is $D = \frac{57.6}{3.14}$ = 18.34394904 inches. .

If we round this off to the nearest tenth of an inch, we get D = 18.3 inches.

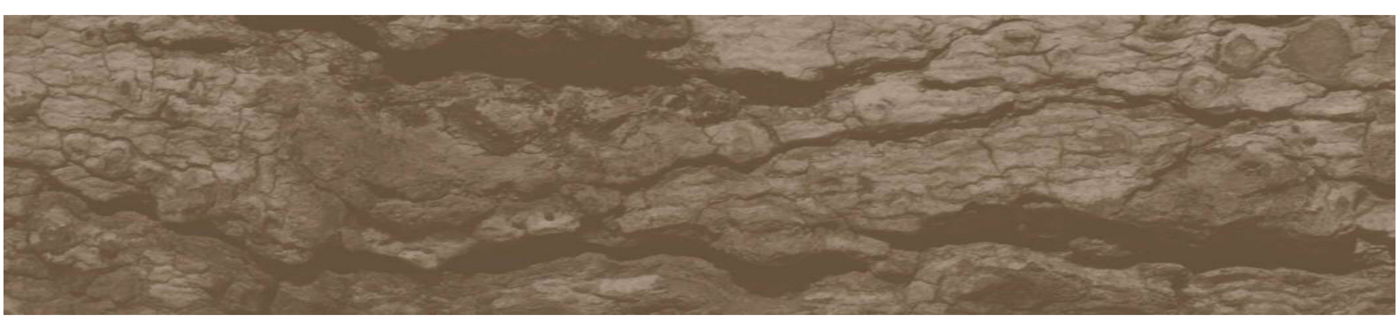
Enter the Diameters of trees #2 — 10 in **Table 3**.

Table 3

Tree #	1	2	3	4	5	6	7	8	9	10
Diameter (in inches)	18.3									

Use the measurements in Table 3 to calculate the *average diameter* of the ten trees:

Average Diameter = $\frac{189.5}{10}$ = ______________ inches



CALCULATING BOARD FEET IN A STANDING TREE

The volume of lumber is calculated in board feet. A board foot is an amount of wood measuring one foot by one foot by one inch. In this activity, let's assume that we have already measured the diameter of ten trees. Since these trees are being measured for logging, the height of these ten trees is measured up to the minimum merchantable log diameter, where the tree is approximately 8 top 10 inches wide at the top. These measurements are recorded in **Table 4**.

Table 4

Tree #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Height **(in feet)**	45	52	57	60	55	49	61	55	59	57

With this information, we can perform the following steps in order to determine the board feet available in each standing tree.

1. Calculate the Area of the surface of a theoretical slice of the tree from the diameter measured at chest height. (see Table 3)

Area = 3.14 multiplied by the radius squared
(or Area = (diameter/2)2 x 3.14 square inches)

Example: Tree #1 has a diameter of 18.3 inches. The radius is 9.15 inches. Applying the formula:

9.15 squared is 83.7
83.7 multiplied by pi (3.14) = **262.9 square inches**

2. Convert the area in square inches to square feet by multiplying as follows:

Area (*in square inches*) x .00694 = Area (*in square feet*)

Tree #1: 262.9 square inches multiplied by .00694 = **1.8 square feet**

3. You can now calculate the potential Cubic Feet of lumber in the tree by using the following formula:

Cubic Feet of Lumber in the Tree = Area (*square ft.*) x Height (*ft.*)/4
(This formula was developed by the wood processing industry to determine the cubic feet of wood from a mathematically idealized tree.)

Tree #1: 1.8 square feet multiplied by 45 (*the height of the tree from Table 4*) = 82
Now, divide 82 by 4 and you get **20.5 cubic feet of lumber**.

The last step is to convert cubic feet of lumber to board feet.

Board Feet = Cubic Feet x 12 (*Cubic Feet multiplied by 12*)

Tree #1: 20.5 cubic feet of lumber multiplied by 12 = **246 board feet of lumber**

This value is added in **Table 5**. Complete Table 5 by doing these calculations for Trees 2 through 10. Record the number of potential board feet of lumber from each tree in **Table 5**.

Table 5

Tree #	1	2	3	4	5	6	7	8	9	10
Board Feet	246									

Using the values in **Table 5**, calculate the average board feet of the trees, rounding to the nearest board foot. (*add the total board feet and divide by the number of trees.*)

Average Board Feet = ________________________________

CHALLENGE ACTIVITIES:

Activity #1: Use this method to calculate the board feet of lumber in a tree of known height with a top taper of 8 to 10 inches. (*See **Geometry Lesson 1** for a method of determining the height of a tree mathematically.*) Or if you like, use this method to calculate the board feet in a log lying on the ground.

Height of the Trees: __________________

Area of Surface: __________________

Square Feet of Surface: ____________________

Cubic Feet: ___________________

Board Feet: ___________________

Activity #2: Select a specific tree. Calculate the board feet as in **Activity 1**. Then determine the specie of the tree. Do research on the internet to determine the current value of a board foot of wood for that specie of tree. Then calculate the current dollar value of the wood in the tree.

Activity #3: Calculate the economic value of the wood in a stand of trees.

Algebra Lesson 3

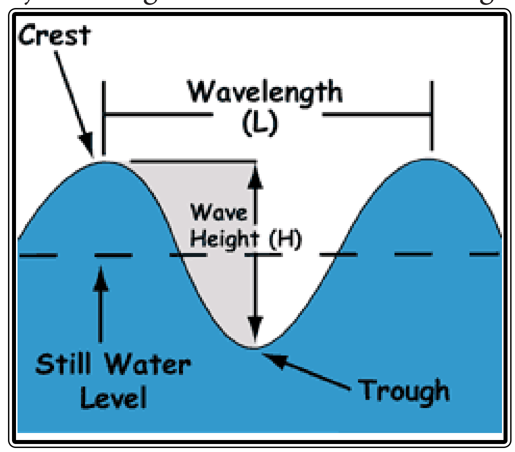
Tlingit Phrase: Óoxjaa tóox̲ yaa kak̲úx.

English Translation: It (boat) is travelling in a wind.

Lesson 3 - Measuring Wavelength

The ocean is part of the natural world around us. It's part of our daily lives, even though we may not always notice it. Have you ever been riding to school, looked out into the channel and thought, "Wow, it's really rough out there today," or "Man, it's glassy calm"? These are typical ways that we might describe the ocean.

If you are on the phone to a friend in Dutch Harbor, and you say that the ocean is rough today, you have an image in your mind of what you mean. Your friend might interpret your statement with an entirely different image. Maybe your friend is a fisher in the Bering Sea, so rough conjures images of fierce waves breaking over the bow of the boat. The same description – rough – might be interpreted by a person living on a lake in California as two foot chop, with occasional white spray blowing off the crests of the waves.



Descriptions like these are **relative** to the person's experiences. They are flawed from the outset because they mean different things to different people. We call descriptions like these **qualitative**, because they describe the qualities a situation has (*big, small, rough, calm, etc.*)

We can convey our meaning more clearly if we quantify the waves we are describing. A **quantitative** description involves assigning numerical quantities to what we are describing. The height of a wave is measured from the trough in front of it to its crest. (*See Illustration*) While, on any given day, waves will have varying heights, the average wave height is a good description of how rough the sea conditions are. To say, "The waves are three and a half feet today," is a description that will mean exactly the same thing to the fisher in the Bering Sea as it will to the person beside the lake in California. In this way mathematical descriptions of our world translate across language and experience barriers.

Another way we can accurately describe a wave is to compare it to nearby waves. "That wave is twice as big as the one before it," would paint a good picture, or "It seems like every seventh wave is about two feet higher than the others." If we know the size of the wave we are comparing to, a picture of the surrounding waves would come into focus. This way, we can express each succeeding wave relative to the size of the first wave.

Activity 1

Let's try to write a quantitative description of the waves rolling in at a beach in Southeast Alaska. While on the beach, you begin to notice a pattern in the waves crashing on shore. For each wave, write an algebraic expression for its size in terms of the first wave. We will say that the first wave's height is x feet.

Wave # and Description	**Algebraic Expression**
1. x feet high	x
2. 1 foot more than the first	$x + 1$
3. 1 foot less than the first	$x - 1$
4. The third wave, decreased by 1 foot	$x - 2$
5. Twice as big as the fourth wave	$2(x - 2)$
6. Three feet more than the fourth	$x - 2 + 3$ or $x + 1$
7. 1 foot more than twice the fourth	$2(x-2) + 1$
8. 2 less than the sum of the last two waves	$(x + 1) + 2(x - 2) + 1 - 2$

To find the **average** wave height, we add all the heights together and then divide by the number of waves. On the space below, write an algebraic expression for the average wave height. Use the algebraic expressions you wrote above to find the expression for the average wave height.

Simplify the numerator by combining like terms. The result is a formula for determining the Average Wave Height.

Average Wave Height: $\frac{\_\_\_\_\_\_\_\_\_\_\_\_}{8}$ =

(Circle the Average Wave Height formula. You will use it soon.)

The wind is now blowing at <u>20 mph</u> and the height of the first wave is 3 feet. List the heights of the eight waves using your eight algebraic expressions with x = 3.

1. ______________ 5. ______________

2. ______________ 6. ______________

3. ______________ 7. ______________

4. ______________ 8. ______________

Calculate the Average Wave Height by using simple arithmetic. That is, add each of the wave heights and divide the sum by 8.

____
Average Wave Height

Now, use the algebraic formula to find the Average Wave Height. (Remember, $x = 3$)

Calculate the Average Wave Height by using simple arithmetic. That is, add each of the wave heights and divide the sum by 8.

____
Average Wave Height

Do your answers agree? Yes _____ No _____

The wind picks up from 20 mph to 30 mph. You notice that the same pattern persists, but all the waves have increased in size. Now the first wave is 4 feet high. ($x = 4$)

List the sizes of the eight waves.

1. _______________ 5. _______________

2. _______________ 6. _______________

3. _______________ 7. _______________

4. _______________ 8. _______________

Calculate the Average Wave Height arithmetically using your list.
(*Add all eight wave heights and divide by 8.*)

____
Average Wave Height

Now, calculate the Average Wave Height with algebra using the Average Wave Height formula. ($x = 4$)
Do your answers agree? Yes _____ No _____

How does this answer compare with your first average?

Answer: ____

If the wind increased from 30 mph to 40 mph, what do you suppose the average wave height would be?

Answer: ____

Let's check to see if you are correct. When the wind blows 40 mph, the size of the first wave is 5 feet. Use any method to calculate the Average Wave Height.

____
Average Wave Height

Write a statement about the relationship between wind speed and the average wave height at the beach. Try to give a quantitative description.

____

____

____

____

____

Noticing patterns is the key to understanding the world around us. Mathematics allows us to describe nature in terms that give everyone the same picture.

In the last activity, we identified a relationship between the size of the waves and the speed of the wind. The size of the waves *depends* on the speed of the wind. This is a **cause and effect** relationship. Wind is the cause, and waves are the effect. The reverse would not be true. The speed of the wind does not depend on the size of the waves.

In this relationship, wind speed is an **independent variable** and wave size is the **dependent variable**. Another way of stating this *dependent-independent relationship* is to say "the size of the waves is a *function* of the speed of the wind." The expression "is a function of" means "depends on."

Other examples of dependent-independent relationships are: "The number of *pounds* of turkey you eat at Thanksgiving is a function of how hungry you are," and "The distance *you* travel on your bicycle in 30 minutes is a function of how fast you peddle."   Describing a relationship quantitatively will be our goal in the next exercise.

Activity 2

At the end of Activity 1 you described the relationship between wind speed and the average wave height at a beach. One possible description might be, "*As the wind speed increases, the waves get bigger*." A more precise description might be, "*Every time the wind increases by 10 mph, the size of the waves increase by 1.5 feet.*"

We could describe this relationship even more precisely by using an equation: $H = 0.15w$, where H = the height of the waves in feet, and w = the wind speed in mph.

- When $w = 20$, $H = 3$ exactly as we saw in the last exercise.
- Also, when $w = 30$, then we can find H by substituting 30 into the equation: $H = 0.15(30) = 4.5$. This also agrees with what we saw at the beach.
- Each of these can be considered as a w-input, along with an H-output, and can be written as an **or dered pair**, (w, H). The ordered pairs would be (20, 3), and (30, 4.5).
- **Important point**--in an *ordered pair* like (w, H), the first coordinate is the **independent variable**, and the second coordinate is the **dependent variable**.

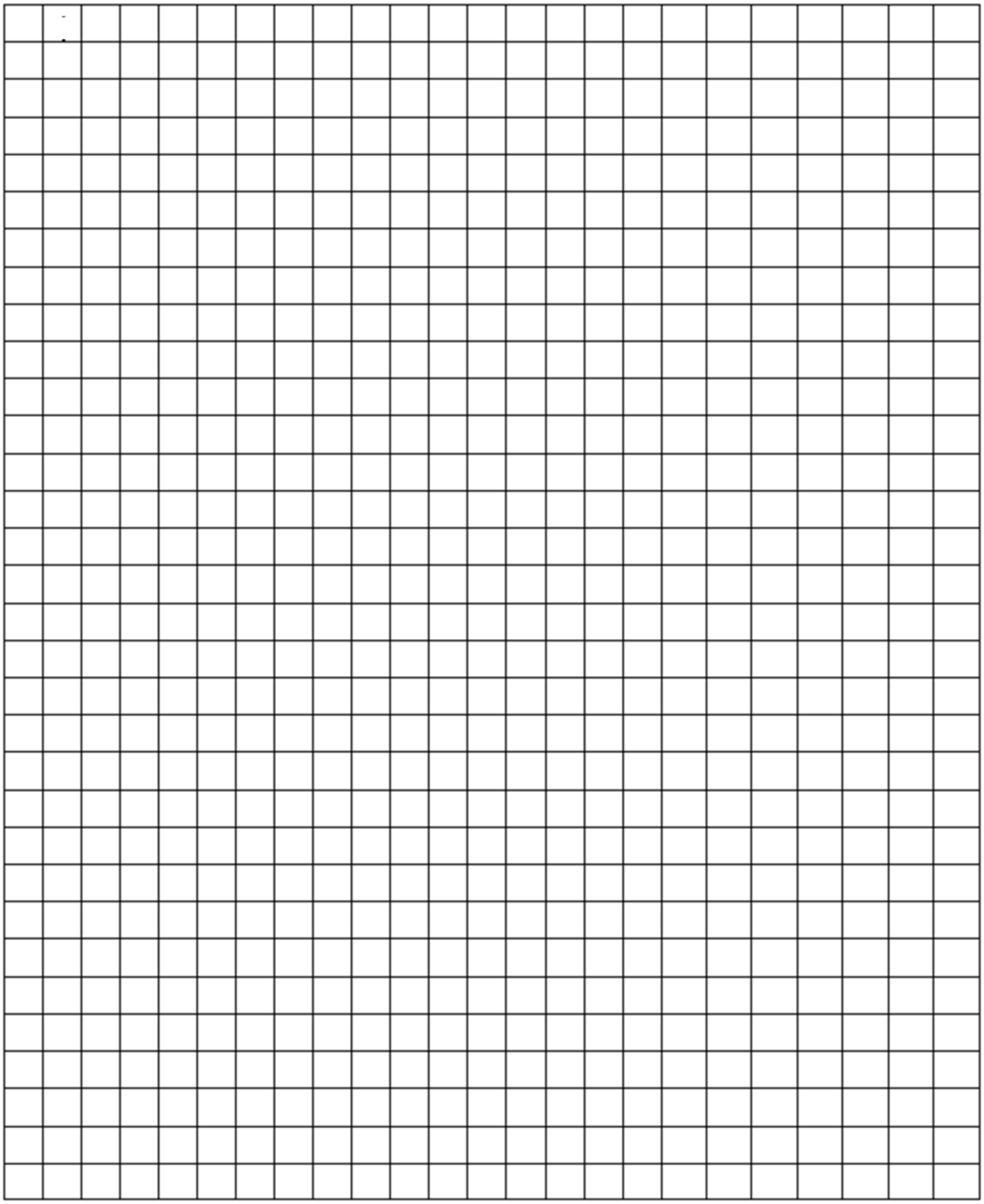
Use the equation $H = 0.15w$ to fill out the table of values and find some ordered pairs:

w input	H output	(w, H) ordered pair
0		
10		
20	3	(20, 3)
30	4.5	(30. 4.5)
40		
50		

Each of these ordered pairs can then be plotted on a graph to show the height of the waves, H, as a function of wind speed, w. Graph the rest of the ordered pairs and draw the graph:

On a graph, the horizontal coordinate is always the independent variable (*w in this case*) and the vertical coordinate is always the dependent variable (*H in this case*). Now, graph these ordered pairs on graph paper. On your graph, extend the order pairs out to include a wind speed of 80 mph.

(DEPENDENT VARIABLE)



INDEPENDENT VARIABLE

Algebra Lesson 3 Extension

Cause and effect relationships surround us every day. Make three statements showing a dependent-independent relationship. You can use your imagination with this, but keep in mind that you must try to quantify one of these relationships:

1. "______________________ is a function of ________________________"

2. "______________________ is a function of ________________________"

3. "______________________ is a function of ________________________"

Choose one of your ordered pairs. Make a table of values to get several ordered pairs.

Input	Output	Ordered Pair

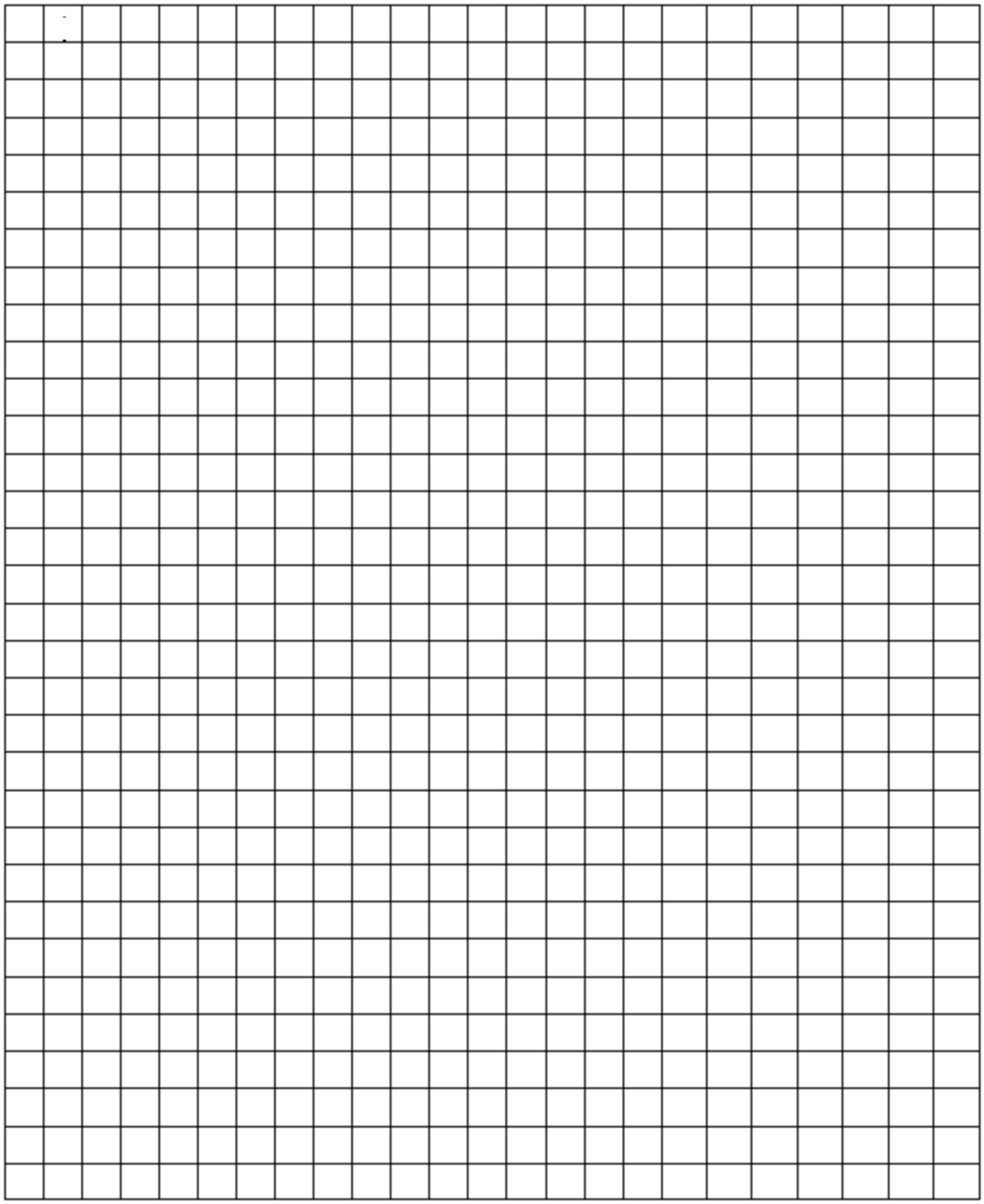
What is the independent variable? ______________________________

What is the dependent variable? ______________________________

Write an equation describing the relationship:

____

Now, graph the relationship on the graph paper. Label the graph correctly.



(DEPENDENT VARIABLE)

INDEPENDENT VARIABLE

Algebra Lesson 4

Tlingit Phrase: Shukalx̱aach.

English Translation: S/he's trolling.

Lesson 4 - Commercial Fishing Business Simulation

Imagine that you are the owner and captain of a fishing boat. In addition to being an experienced fisher, you must be a competent business person. Each fishing season you must meet your annual business expenses. These expenses begin to accumulate even before the fishing season begins. Pre-season expenses may include the boat payments, cost of travel, insurance, groceries, and boat maintenance. During the fishing season, major expenses include fuel, pay for crew, groceries, and unexpected maintenance. And every day you hope that your skill, experience, and good luck allow you to catch enough salmon to pay your expenses and make a profit. Let's do several exercises involving the business of salmon fishing.

Activity 1

Let x = average weight of one fish.

On the first set:

4 fish of average weight	4x
2 fish that are 1 lb. less than average	2(x-1)
3 fish that are 2 lbs. over the average	3(x+2)

The total weight is 49 lbs.

Solve $4x + 2(x-1) + 3(x+2) = 49$
What is the average weight of one fish?

Answer:

Activity 2

Here are the anticipated **expenses** for the fishing season:

Boat Payment	$9,000
Fuel	$4,500
Maintenance	$8,000
Food	$1,600
Crew Pay	$32,000
Insurance	$2,300
Travel	$2,100
Miscellaneous	$3,500

Calculate the total expenses.

Now, let's calculate total revenue we will need to break even—that is, to pay our **expenses** for the season. Let x = the number of lbs. of fish caught. We are going to fish exclusively for sockeye this season. Based on our experience over the last several years, we expect the price of sockeye to average $1.40 per pound this season. Given this price per pound, find the number of pounds of sockeye you will need to catch to break even.

Activity 3

During a single 12-hour opener, a total of 231,000 fish were caught by two hundred twenty-seven fishing boats. An average fish weighed 6.3 pounds. Find the average catch per boat by weight and price.

We caught 1238 fish. Did we miss, meet, or exceed the average?

How many days must we average this catch rate to break even?

Activity 4

Mike has a load of fish to drop off at the tender, which is three miles away. Mike is traveling at a speed of eight knots. How long will it take Mike to reach the anchored tender? Time = Distance divided by Rate ($T = D/R$)

Answer:

On another day Mike is traveling to a tender 5 miles away. The tide is in full flood and Mike is bucking a 3 knot current. Mike has the boat running at a water speed of 10 knots. How long will it take for Mike to reach the tender? Round the answer to the nearest minute.

Answer:

On another day Mike is traveling to a tender 5 miles away. The tide is in full flood and Mike is bucking a 3 knot current. Mike has the boat running at a water speed of 10 knots. How long will it take for Mike to reach the tender? Round the answer to the nearest minute.

Answer:

Now let's say that the tide is in full ebb and Mike and Dave are heading toward an anchored tender. Mike is traveling with the tide, which is running at 4 knots. He is traveling at 9.5 knots speed over the water. Dave is headed against the tide and is traveling at 12 knots over the water. If Mike is 7 miles from the tender and Dave is 4 miles from the tender, who will get there first and by how many minutes? (*Round to the nearest minute.*)

Answer:

Activity 5

After delivering the catch to the tender, we fuel up. Reading the Hobbs meter at fill up, we learn that we have used 245 gallons of diesel during three days of fishing. At the previous fill up, the boat took on 223 gallons of diesel after two days of fishing. Given the total gallons of fuel, what is our average fuel consumption rate per day?

Answer:

If we burn this amount of fuel for the days needed to break even, and the cost is $4.39 per gallon, will this agree with our fuel cost estimate? (*In other words, is our estimated cost of fuel to break even too low, right on, or too high?*)

Answer:

Activity 6

By the end of the season, we caught 94,650 pounds of sockeye salmon at a fixed price of $1.40 per pound. We also incurred the following expenses: (*See Boat Payment and Insurance in Exercise 2.*)

Crew Pay	25% of gross (i.e. total revenue)
Travel	$1,745
Diesel fuel	$5,986
Maintenance	$5,750
Food	$1,825
Miscellaneous	$1,645

How much profit did we make after expenses?

Answer:

Algebra Lesson 5

Tlingit Phrase: Kei k̲ugux̲sa.áat´.

English Translation: It will be cold.

Lesson 5 - Glacier Calculations

Southeast Alaska is a land of glaciers. Our maritime climate and coastal mountains create the conditions which favor the growth and ice fields and glaciers. There are over 100,000 glaciers in Alaska. Over half of the earth's mountain glaciers are found in Alaska. Glaciers cover five percent of the state.

Glaciers are dynamic rivers of ice which have sculptured the mountains and valleys of Southeast for thousands of years. As a glacier moves forward, or advances, it is also melting. If the melt rate exceeds the rate of advancement, we say the glacier is receding. Today, most of the glaciers in Southeast are receding. One of the most dramatic examples of a receding glacier is located in the Mendenhall Valley near Juneau.

The Juneau Ice field, which receives over 100 feet of snow each year, is the birthplace of 38 glaciers. The most notable of these is the Mendenhall Glacier. As America's most visited glacier, the Mendenhall Glacier attracts over 500,000 visitors each year. The Mendenhall Glacier is truly magnificent. At over 12 miles long and over a half mile wide at the terminus (*face*), this river of ice is an awesome sight.

Let's examine the dynamic changes in the Mendenhall Glacier using mathematics.

Activity 1

CALCULATING THE RATE OF ICE LOSS

The Mendenhall glacier is receding. This doesn't mean that the glacier is moving backwards. It means that pieces (*icebergs*) are breaking and falling into Mendenhall Lake faster than the glacier is moving forward. Given the rate at which it is receding, we will calculate how fast ice is breaking off the glacier.

In order to calculate the rate ice is breaking off the glacier we need to subtract the total loss of ice from the flow rate. The first step is finding the flow rate from the formula:

Flow rate = distance/time

After measuring the glacier's movement over the period of one day we find it has moved 6 inches, which is .5 feet. When we express this in scientific notation it is:

$$Flow\ rate = \frac{5*10^{-1}\ ft}{1*10^{0}\ day} = \frac{5*10^{-1}\ ft}{1\ day} = 5*10^{-1}\ ft\ /\ day$$

Notice that the zero exponent, 10^0, is equal to 1. This is because the exponent of a number tells us to multiply 1 by the number as many times as the exponent says. For example: = (1) (10) (10) (10) = 1000. However, with zero exponents 1 isn't being multiplied by anything, therefore any number, n, with a zero exponent, $n^0 = 1$.

We also know that the glacier has lost 382.52 feet of ice over the past year, but we need to find the average ice loss per day.

$$Average\ Ice\ loss\ per\ day = \frac{annual\ ice\ loss}{days\ in\ a\ year}$$

When we express this in scientific notation we have:

$$\text{Average ice loss per day} = \frac{3.8252*10^{2}}{3.65*10^{2}} = \frac{3.8252}{3.65}*\frac{10^{2}}{10^{2}} = 1.048\ ft\ /\ day$$

Now we can subtract the total loss of ice from the flow rate:

$$Average\ Ice\ loss\ per\ day = \left(5*10^{-1}\ ft/day\right) - \left(10.48*10^{-1}\ ft/day\right) = -5.48*10^{-1}\ ft/day = -0.548\ ft/day$$

Our result is negative because the average loss of ice per day (1.048 ft / day) is greater than the flow rate of the glacier ($5*10^{-1}$ ft / day). Therefore, the average recession rate of the glacier is -0.548 ft / day. So, even though the glacier moves forward 182.5 ft / year, because it loses 382.52 ft / year of ice we have a recession of -200 ft / year. To check our calculation we multiply the average daily recession rate with the total number of days in the year:

$$\frac{-1.048\ ft}{1\ day} * \frac{365\ days}{1\ year} = \frac{-1.048*365}{1*1} * \frac{ft}{year} * \frac{days}{day} = \frac{-200}{1} * \frac{ft}{year} * 1 = -200\ ft/year$$

Our answers are in agreement, so our calculations are correct.

Now you try it! Using the formulas above, calculate how fast ice is breaking off of a hypothetical glacier.

Glacier A: Ice flow rate is 3 inches a day, and the yearly ice loss is 175 ft.

$$Flow\ rate = \frac{\_\_\_\_\_\_\ ft}{day} = \_\_\_\_ * \_\_\_\_ * \_\_\_\_ = \qquad ft/day$$

$$Average\ ice\ loss\ per\ day = \frac{\_\_\_\_\_\_\ ft}{day} = \_\_\_\_ * \_\_\_\_ * \_\_\_\_ = \qquad ft/day$$

$$Average\ ice\ loss\ per\ day = (\qquad\qquad) - (\qquad\qquad) = \qquad *10^{-1}\ ft/day = \qquad ft/day$$

$$\frac{\_\_\_\_\ ft}{day} * \frac{\_\_\_\_\ days}{year} = \_\_\_\_\_\_ * \_\_\_\_ * \_\_\_\_ = \_\_\_\_ * \_\_\_\_ * 1 = \qquad ft/year$$

Our answers are in agreement (*rounded to the nearest whole digit*), so our calculations are correct.

Activity 2

Given the dimensions of an iceberg visible above the surface, we will calculate an algebraic expression for estimating the total volume. (*Multiplication of Polynomials.*)

The density of pure ice water is 920 kg/m^3 and the density of seawater is 1025 kg/m^3, which is a ratio of approximately 8/9. Therefore, only 1/9th of the total mass of an iceberg is visible above the surface. Given this information, we only need to measure the visible shape of an iceberg to estimate its total volume. To simplify this process we use the volume formula for a pyramid (Area of the Base * Height * 1/3). Remember that the base equals length multiplied by width. (*h = height, B = base*)

$$V_{visible} = 1/3\,hB, \text{ where } V_{visible} = \text{visible volume}, h = height, B = base$$

After substituting the variables $h = x, B = 2xy + 1$ in the formula our equation now is:

$$V_{visible} = 1/3x(2xy+1)$$

Since the total mass of the iceberg is proportional to the total volume, and we know that only 1/9th of the iceberg is visible, we will calculate the total volume by multiplying the visible volume by a factor of 8.

$$V_{total} = 8[1/3\,x(2xy+1)]$$

When we simplify the expression the resulting equation is:

$$V_{total} = 2/3x^2y + 1/3x$$

The dimensions given for our iceberg are: x = 25 ft, y = 30 ft, so our equation yields:

$$V_{total} = 2/3\left[625(30)\right] + 1/3(25) = 12500 + 8.33 = 12508.33 \text{ cu ft}$$

What would the total volume be if x = 50, y = 67?

V_{total} = 2/3 + 1/3 = + = cu ft

What if we set $h = x^2, B = (3x+1)(y-1)$?

What does our equation for the total volume of an iceberg look like now?

V_{total} =

Simplifying the expression yields:

V_{total} =

If we set x = 5, y = 7, what value does our formula yield?

V_{total} =

Activity 3

Given a specific quantity of icebergs, we will calculate an expression for the total volume of ice breaking off over a specific period of time (*adding polynomials*).

Using the formula (V=1/3hB) from **Exercise #3** for the volume of our icebergs, we need to determine how many icebergs have calved from the glacier over a given period of time. After counting and measuring the icebergs that have calved in two days, we find that there are 3 icebergs with varying dimensions:

Iceberg #1: h = x B = (x + y) y

Iceberg #2: h = x B = x (y + 1)

Iceberg #3: h = x B = (x - y) (y - 2)

Inserting the dimension variables for each iceberg generates a polynomial formula for each volume, which we then simplify:

Iceberg #1 = $1/3x[(x+y)y] = 1/3x^2y+1/3xy^2$

Iceberg #2 = $1/3x\ [x(y+1)] = 1/3x^2y+1/3x^2$

Iceberg #3 = $1/3x[(x-y)(y-2)] = 1/3x^2y+(-2/3)x^2+(-1/3)xy^2+2/3xy$

Our equation will take the sum of the simplified polynomials for each iceberg volume multiplied by the period of time for our observations, which is two days:

Volume*Time = $[1/3x^2y+1/3xy^2) + (1/3x^2y+1/3x^2) + (1/3x^2y+-2/3x^2+-1/3xy^2+2/3xy\}]2$days

Simplified, this yields our formula for the total volume of ice lost over a period of two days:

$$V_{total} = 2x^2y + (-0.67)x^2 + 1.33xy \text{ cu.ft}$$

What value does our formula yield if we set x = 30, y = 40?

$$V_{total} = \qquad + \qquad + \qquad =$$

Now you try it! Given the dimensions for icebergs #4, #5, and #6, calculate the total volume of ice breaking off over a period of 5 days using the simplified formula .

Iceberg #4: $h=x^2$ B = (x + 1) y

Iceberg #5: $h=x^2$ B = x (y + 2)

Inserting the dimension variables for each iceberg generates a *polynomial formula* for each volume, which we then simplify:

Iceberg #6: $h=x^2$ $B = x(y - 2)$

Our equation will take the sum of the simplified polynomials for each iceberg volume multiplied by the period of time for our observations, which is 5 days:

Volume*time =

Simplified, this yields our formula for the total volume of ice lost over a period of 5 days:

$$V_{total} =$$

What value does our formula yield if we set ?

$$V_{total} =$$

Additional Information about the Mendenhall Glacier

The Mendenhall Glacier formed the Mendenhall Lake as the glacier receded in the early part of the 20th century. As the glacier melts into the lake, the rate of recession has accelerated. According to the scientists who monitor the rates of glacier recession in Alaska, the recession rate of the Mendenhall Glacier will slow down once the glacier has receded above the lake. For those of us who appreciate the mighty Mendenhall Glacier and the half million tourists who visit this site annually, this is good news.

Geometry Lesson 1

Tlingit Phrase: Ligéi.

English Translation: It is tall.

Lesson 1 - How High is it?

We live in one of the most dramatic landscapes on earth. Thousands of visitors come to Southeast Alaska each year and marvel the majestic landscape. The ice field above Juneau has been described by visitors as a cathedral of rock and ice. We, who live here year-round, are accustomed to seeing tall trees, glaciers, and mountains. As we move from one place to another, whether on foot, by car, boat or small airplane, we constantly orient ourselves by monitoring the visual cues from the landscape.

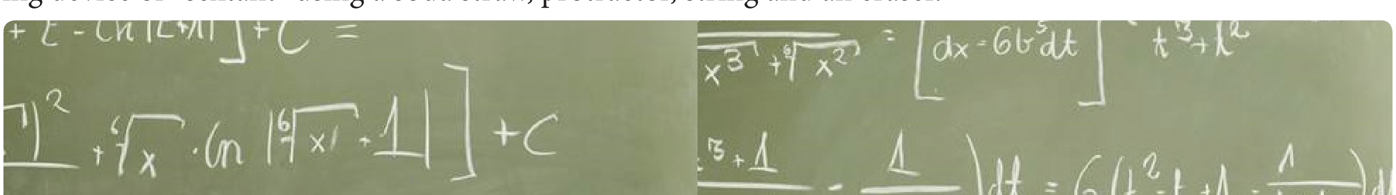
The communities in Southeast are located along fjords and in bays surrounded by mountains. Flat land is a limited resource in Southeast. An Alaskan who is abruptly thrust into totally flat environment may experience a sense of disorientation and confusion. The seemingly unchanging typography may appear boring and featureless to one who is accustomed to the dramatic variations provided by mountains and glaciers. A Southeasterner may experience difficulty determining his or her location and finding direction in an area where there is little change in elevation. A Southeast fisher who fished for several days off the coast near Virginia Beach, Virginia, reported that he finally realized the need for lighthouses on the East Coast. The coastline is virtually flat and featureless from a mile to two offshore. The only discernible features to the mariner were the lighthouses.

Have you ever wondered how tall a large tree actually is, or how tall that mountain across the bay is, or how tall a building is? One way of determining height is to simply measure the height with a tape measure. This is simple to do with small object such a post or single story building. But try to directly measure the height of tall tree, eight story building, or a mountain! Direct measurement may be impossible. That is where math becomes very useful. We can measure the height of tall objects quite accurately by indirectly by using mathematics. Here is how we can do it.

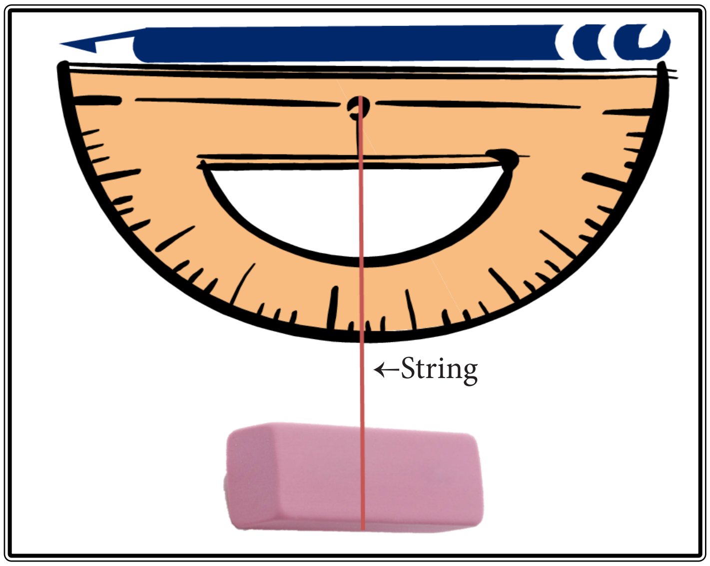
Historical Math Fact: Indian mathematician Radhanath Sikdar was the first person to identify Mount Everest as the tallest mountain on earth. Using the method we are about to learn, he accurately measured the altitude of Mount Everest from India, 150 miles away from the mountain. Early calculations of the height of the mountain led to an interesting conclusion. In the early 1950's a British expedition went to Nepal to determine the exact altitude of Mount Everest. The altitude was declared to be exactly 29,002 feet above sea level. When the expedition returned to Britain, the scientists admitted that the actual calculations revealed that Mount Everest was exactly 29,000 feet tall. The mathematicians arbitrarily added two feet to the official altitude of Mount Everest to avoid giving the impression that the height determined by their expedition was nothing more than an estimate.

Height (*altitude*) calculations are actually easy to make. **First, you will need to determine the baseline–that is the distance from your position to the base of the object that you are measuring**. For an object such as a tree or a building, simply use a tape or other measuring device and determine the distance. For a mountain, you can determine your location and your distance from the center of the base of the mountain using a map.

The second step is to measure the angle of from where you are standing to the top of the object you are measuring. You will need a measuring device to measure this angle. You can make a simple angle measuring device or "sextant" using a soda straw, protractor, string and an eraser.



Activity 1



The angle is found by subtracting the reading from 90 degrees. With your "sextant," determine the angle to several tall landscape features (*such as mountains, tall trees, buildings, flagpole, etc.*) which you can clearly see and measure. Remember: determine the angle by subtracting your reading from 90 degrees. Suggestion – do this in pairs. One person sights the sextant and the other person reads the angle. Record your reading below:

Feature Being Measured	**Angle**
______________________________	___________
______________________________	___________
______________________________	___________
______________________________	___________
______________________________	___________

The third step is to consult a Tangent Table and find the Tangent of the angle.

Angle	tan(a)
0.0	0.00
1.0	.0175
2.0	.0349
3.0	.0524
4.0	.0699
5.0	.0875
6.0	.1051
7.0	.1228
8.0	.1405
9.0	.1584
10.0	.1763
11.0	.1944
12.0	.2126
13.0	.2309
14.0	.2493
15.0	.2679
16.0	.2867
17.0	.3057
18.0	.3249
19.0	.3443
20.0	.3640
21.0	.3839
22.0	.4040
23.0	.4245
24.0	.4452

Angle	tan(a)
25.0	.4663
26.0	.4877
27.0	.5095
28.0	.5317
29.0	.5543
30.0	.5773
31.0	.6009
32.0	.6249
33.0	.6494
34.0	.6745
35.0	.7002
36.0	.7265
37.0	.7535
38.0	.7813
39.0	.8098
40.0	.8391
41.0	.8693
42.0	.9004
43.0	.9325
44.0	.9657
45.0	1.000

Angle	tan(a)
46.0	1.0355
47.0	1.0724
48.0	1.1106
49.0	1.1504
50.0	1.1918
51.0	1.2349
52.0	1.2799
53.0	1.3270
54.0	1.3764
55.0	1.4281
56.0	1.4826
57.0	1.5399
58.0	1.6003
59.0	1.6643
60.0	1.7321
61.0	1.8040
62.0	1.8907
63.0	1.9626
64.0	2.0503
65.0	2.1445
66.0	2.5460
67.0	2.3559
68.0	2.4751
69.0	2.6051
70.0	2.7475

Angle	tan(a)
71.0	2.9042
72.0	3.0777
73.0	3.2709
74.0	3.4874
75.0	3.7321
76.0	4.0108
77.0	4.3315
78.0	4.7046
79.0	5.1446
80.0	5.6713
81.0	6.3138
82.0	7.1154
83.0	8.1443
84.0	9.5144
85.0	11.430
86.0	14.301
87.0	19.081
88.0	28.636
89.0	57.290
90.0	INFINITE

The fourth and final step is to determine the height by multiplying baseline by the tangent of the angle. The product is the height of the object you are measuring. For example, you are 10 meters away from a flagpole and you have determined the angle is 45 degrees. The tangent of 45 degrees is 1.0. Multiplying 10 by 1.0 you determine that the height of the flagpole is 10 meters.

Why does this work? What exactly is the tangent? A tangent is a function. In math, a function is a relationship between two numbers. For example, let us say that you are buying several cans of soup at the local store. Each can, X, costs $1.85. Let us assume that the variable Y is the total cost you will pay for the number of cans you buy. The function, or relationship between these two variables, can be expressed as:

Y = 1.85X

You bought one can of soup. Then Y, the total cost, is $1.85. If you bought two cans of soup, Y = $2.70. This function can be expressed as follows: **The tangent of X is 1.85**
Let's use this function to determine Y (*the total cost*) of several different values of X (*the total number of cans*).

If	X = 3,	then	Y = $5.55
If	X = 4,	then	Y = $7.40
If	X = 5,	then	Y = $9.25

What would be the value of Y if X = 45? ____________________

In math shorthand, this function is written as **tan(X) = 1.85**

The tangent of the angle is a function defined as: $\mathbf{\tan x = \dfrac{\sin x}{\text{cosine } x}}$

Activity 2

With a partner, determine the height of a nearby building.

Follow these four steps:

With a tape measure or other measuring device, measure the distance between yourself and the building.

____

(Distance)

Determine the angle of sight to the top of the building. (Remember to subtract your reading from 90 degrees when using your homemade sextant.)

____

(Angle)

Look up the tangent of the angle in the tangent table.

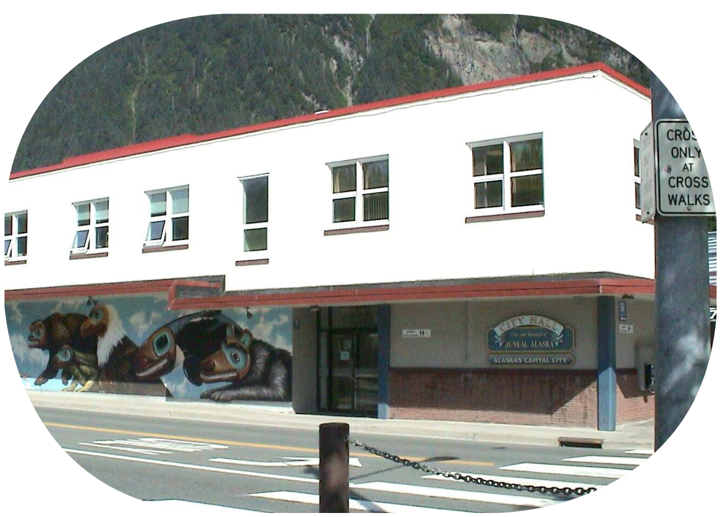
____

(Tangent of the Angle)

Now, multiply the baseline (your distance from the building) by the tangent of the angle.

____

(Height of the Building)



Activity 3

With a partner, use this method to determine the height of four manmade or natural features. If you are measuring the height of a mountain which is a significant distance from you, use a map or some other resource to determine the distance.

1) ______________________ (Object or Geographic Feature)

______________________ (Distance)

______________________ (Angle)

______________________ (Tangent of the Angle)

______________________ (Height)

2) ______________________ (Object or Geographic Feature)

______________________ (Distance)

______________________ (Angle)

______________________ (Tangent of the Angle)

______________________ (Height)

3) ______________________ (Object or Geographic Feature)

______________________ (Distance)

______________________ (Angle)

______________________ (Tangent of the Angle)

______________________ (Height)

4) ______________________ (Object or Geographic Feature)

______________________ (Distance)

______________________ (Angle)

______________________ (Tangent of the Angle)

______________________ (Height)

Geometry Lesson 1 Extension

To better understand *sine*, *cosine* and *tangent*, let's take a look at these triangles. Begin with the definitions. We start with a right triangle. The side opposite from the right angle is the hypotenuse, "**h**". This is the longest side of the three sides of a right triangle

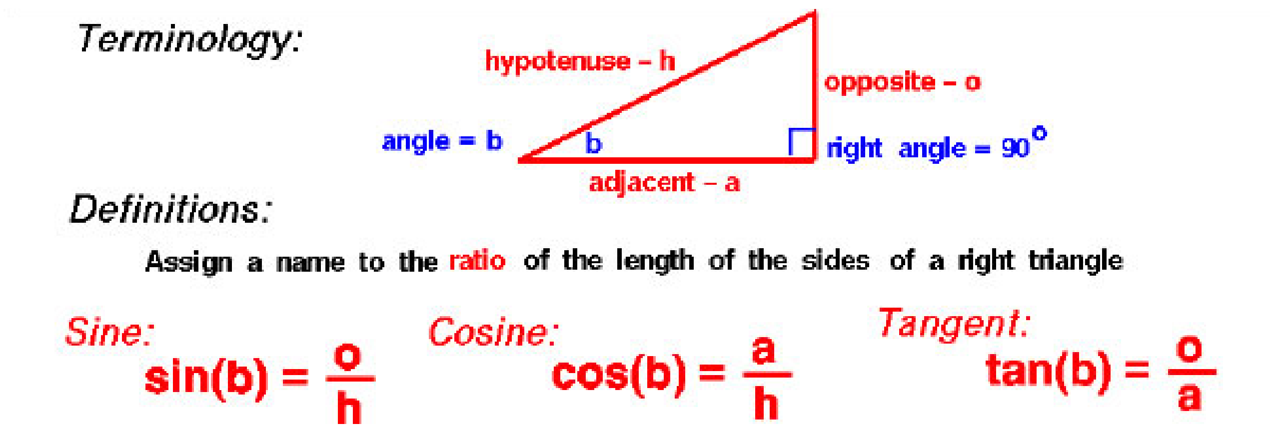


Photo Credit: *NASA, Glenn Research Center*

We pick one of the other two angles and label it **angle b**. Since the sum of all angles of a triangle is 180 degrees, if we know the value of **b**, then we know that the value of the third angle is 90 - **b**. (Remember–a right triangle has one angle that is 90 degrees.) The side opposite, the **angle b**, we will call **o** for "*opposite*". The remaining side we will label a for "*adjacent*". The three sides of the triangle are labeled **o**, **a** and **h**. The sides **a** and **h** make up **angle b**.

The ratio of the right sides of a right triangle depends only on the value of the **angle b**.
We define the ratio of the opposite side of the hypotenuse to the *sine* of the **angle b** and give it the symbol **sin(b)**.

$$\sin(b) = o/h$$

The ratio of the adjacent side to the hypotenuse is called the *cosine* of the **angle b** and given the symbol **cos(b)**. It is called the *cosine* because its value is the same as the *sine* of the other angle in the triangle which is not the right angle.

$$\cos(b) = a/h$$

Finally, the ratio of the opposite side to the adjacent side is called the tangent of the angle b and is given the symbol tan(b).

$$\tan(b) = o/a$$

To demonstrate the value of *sine*, *cosine* and *tangent*, let's look at these three triangles on page 46.

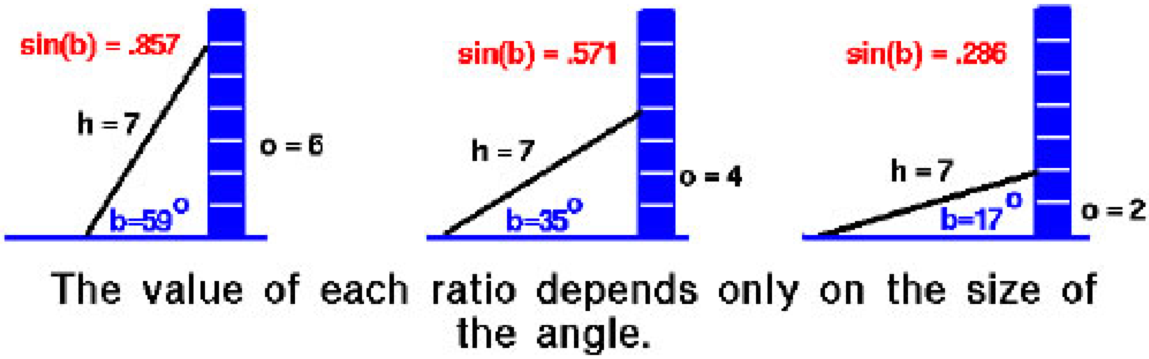


Photo Credit: *NASA, Glenn Research Center*

In the first example, we have a 7 foot ladder that we lean against a wall. The wall is 7 feet high. We have drawn white lines on the wall at one foot intervals. The length of the ladder is fixed. If we incline the ladder so that it touches the 6 foot line, the ladder forms an angle of nearly 59 degrees to the ground. The ladder, ground and wall form a right triangle. The ratio of the height on the wall (*o - opposite*) to the length of the ladder (*h - hypotenuse*) is 6/7, which equals roughly .857. This ratio is defined to be the sine of b = 59 degrees. The ratio stays the same for any triangle with a 59 degree angle.

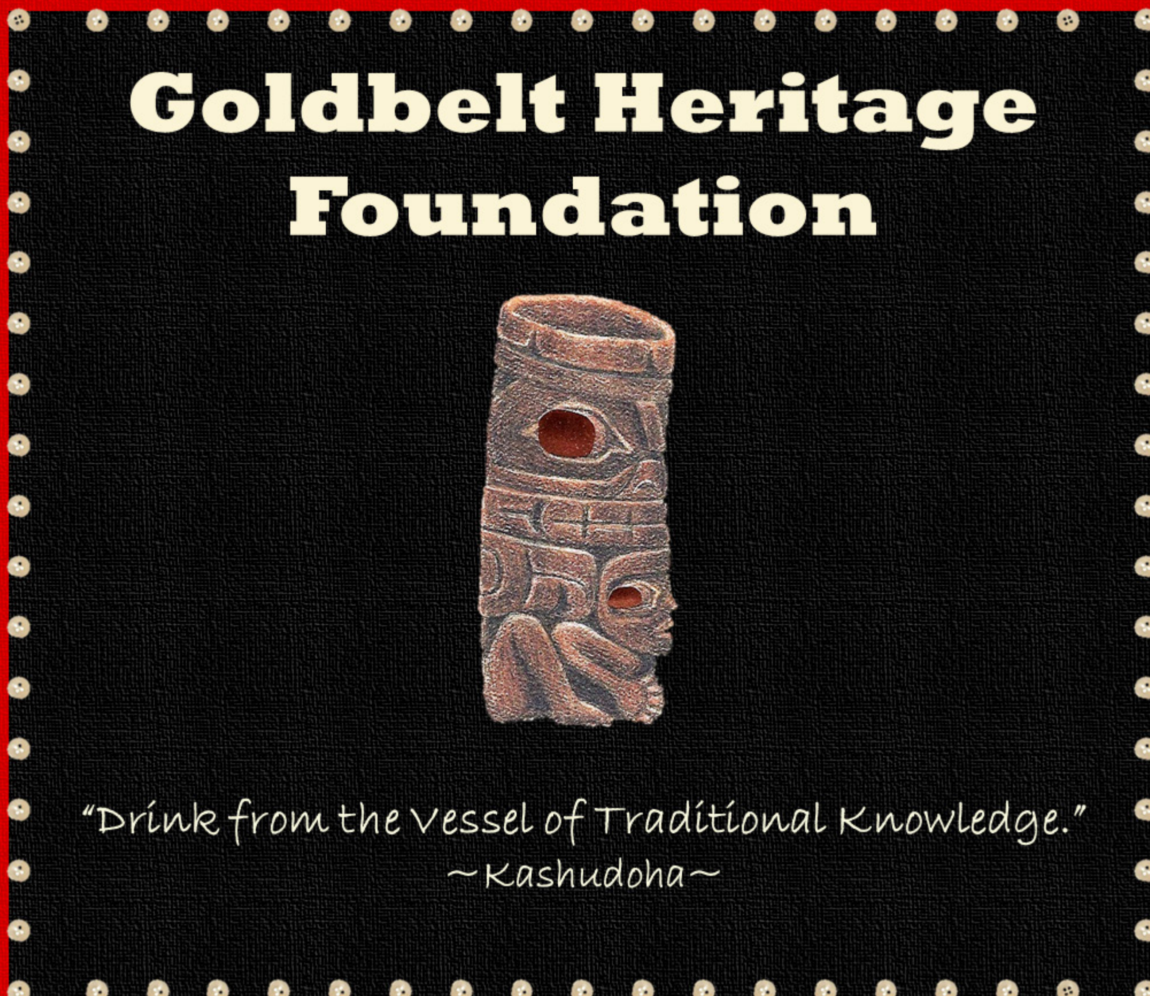
In the second example, we incline the 7 foot ladder so that it only reaches the 4 foot line. As shown in the figure, the ladder is no inclined at a lower angle than in the first example. The angle is about 35 degrees. The ratio, of the opposite of the hypotenuse, is now 4/7, which equals roughly .571.

In the third example, the 7 foot ladder only reaches the 2 foot line. The angle decreases to about 17 degrees and the ratio is 2/7, which is about .286. As you can see, for every angle there is a unique point on the wall that the 7 foot ladder touches. It is the same point every time we set the ladder to that angle. Mathematicians call this situation a **function**.

Since the *sine*, *cosine* and *tangent* are all functions of the **angle b**, We can measure the ratios once and produce tables of the values of the *sine*, *cosine* and *tangent* for the various values of **b**. Later, if we know the value of an angle in a right triangle, the tables tell us the ratio of the sides of the triangle. If we know the length of any one side, we can solve for the length of the other sides. Or if we know the ratio of any two sides of a right triangle, we can find the value of the angle between the sides.

<u>We can use these tables to solve problems in the real world.</u>

Gunalchéesh!

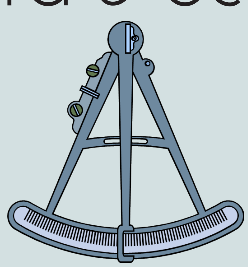


This curriculum was brought to you by **Goldbelt Heritage Foundation**, a non-profit organization that was formed in 2007 to document the Tlingit language and stories to preserve our culture and history for future generations. The Foundation seeks to translate the Tlingit oral language into a written language. Due to the complexity of the Tlingit language, the process involves years of effort and research with Tlingit elders and language specialists.

Southeast Math

Grade Levels 7 – 12

Algebra & Geometry



Goldbelt Heritage Foundation

"Drink from the Vessel of Traditional Knowledge."

This unit was brought to you by the **Demonstration Grant, Award # S299A90070**. The Demonstration grant supports the Tlingit culture and language being taught to Southeast Alaska's Youth. This project develops and disseminates culturally responsive science and math curriculum to Alaska schools.

Southeast Math

7-12 Algebra/Geometry

Credits

Author: Paul Berg
Math Advisor: Ward F. Ward
Photo Credits: Virginia Berg
Editor: Tiffany La Rue

Table of Contents

Teacher Guidelines

ALGEBRA LESSON 1

OBJECTIVES
Students will:

- Find values of unknown quantities by using algebraic expressions
- Solve linear equations for unknown quantities
- Balance linear equations
- Learn to solve for X

TIME 60-90 minutes

MATERIALS Pencil and paper

ACTIVITIES
Activity 1: Determine the weight of an unknown quantity of a fish.
Activity 2: Find the total weight of fish caught during a specific time period.
Activity3: Calculate the quantity of eggs laid by salmon.
Activity 4: Determine the probable loss of eggs due to predation.

ASSESSMENT
Activity 1: Students use an algebraic expression to find an unknown weight.

Activity 2: Students accurately solve expression to find the total weight of salmon in a stream during a 10 day period.

Activity 3: Students solve for an unknown to find out how many salmon eggs will be deposited over a 10 day period.

Activity 4: Students calculate how many fish will be caught by bears during a 10 day period.

OPTIONAL EXPANSION ACTIVITIES
Ask elders about the history of salmon runs in a stream near your home. Note how the salmon runs have or have not changed over time.

Do a stream study. Select a convenient salmon stream. Using a sampling technique, calculate the number of salmon coming upstream for a specific time period (5 to 10 days). Calculate the biomass of the salmon, the number of eggs deposited, and how many salmon/salmon eggs will be lost to predation.

ALGEBRA LESSON 2

OBJECTIVES
- Learn how to calculate the age of a tree by examining the growth rings on the wood
- Calculate a tree's diameter to help determine its age.
- Learn to use pi and expressions for "circumference" and "diameter" to produce accurate measure ments.
- Calculate volume of lumber (by board feet) in each tree.
- Convert measurements from inches to feet.
- Learn how to convert data from tables onto a line plot.
- Calculate averages.

TIME 60 + minutes

MATERIALS
- Pencil and paper
- Graph paper (optional)
- Calculator

BACKGROUND INFORMATION
The Tongass National Forest is three times the size of the next largest national forest in the United States. The rainforest of Southeast has sustained a complex ecosystem for hundreds of thousands of years–yet the system is under strain. Human habitation, the extraction of resources on an industrial scale and warming climate threaten this unique environment. By applying math skills to investigate a stand of trees, students have the opportunity to learn more about this majestic resource as they learn math skills in a realistic setting.

For this lesson, select a stand of trees which have a similar diameter. Trees with similar diameter will have a similar approximate age. Many areas of Southeast have been logged over and have second growth timber of the same approximate age. If you are in an area which has old growth, you can select a stand of old growth forest for your students to study.

ACTIVITIES
- Plotting measurements
- Use of pi
- Calculating averages
- Determining volume, circumference, and diameter

ASSESSMENT
Students will measure approximate diameter of trees
Students will plot measurements of a tree's diameter on graph paper (line graphs)
Students will calculate the volume of lumber produced from one tree.

OPTIONAL EXPANSION ACTIVITIES
1. Have students identify areas of old growth timber, second growth and logged over areas within the local geographic region. Plot these areas on a map or create a color coded map of the area. Calculate the relative percentages of old growth timber, second growth timber, recently logged areas and non-forested lands (i.e. glacier, mountains, wetlands, etc.)
2. Calculate the
board feet of lumber that can be produced from a single tree.
3. Find the area, square footage, and feet of a sheet of lumber produced from a single tree.

ALGEBRA LESSON 2 - TEACHER GUIDE INFO, CONT.

The Tongass National Forest is three times the size of the next largest national forest in the United States. The rainforest of Southeast has sustained a complex ecosystem for hundreds of thousands of years, yet the system is under strain. Human habitation, the extraction of resources on an industrial scale, and a warming climate threaten this unique environment. By applying math skills to investigate a stand of trees, students have the opportunity to learn more about this majestic resource in a realistic setting.

GENERAL INFORMATION

In this math activity, students use math to investigate a simulated stand of trees. This part of the activity can be completed in the classroom. For best results, extend this learning activity to the real world. Select a stand of trees which have a similar diameter. Trees with similar diameter will have a similar approximate age. Many areas of Southeast have been logged and have second growth timber of the same approximate age. If you are in an area which has old growth, you many wish to select a stand of old growth forest for your students to study.

EXPANDING THE LESSON

1. You can also have your students measure the trees by species. Growth rates of various species vary.

2. Have students identify areas of old growth timber, second growth, and logged over areas which can be found within the local area. Plot these areas on a map or create a color coded map. Calculate the relative percentages of old growth timber, second growth timber, recently logged areas, and non-forested lands (i.e. glacier, mountains, wetlands, etc.)

ALGEBRA LESSON 3

ALASKA MATHEMATICS STANDARDS (adopted June 2012)
Grade 8 Math, (1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems, and 8.EE Work with radicals and exponents.
Interpreting Functions (F–IF), Understand the concept of a function and use function notation, and analyze functions using different notations
Building Functions (F-BF), Build a function that models a relationship between two quantities

OBJECTIVES

- Introduce independent and dependent variables and ordered pairs
- Identify the appropriate tool and/or unit of measurement with which to measure objects
- Uses appropriate symbols to justify solutions to mathematics problems
- Converts representations of data between graphs, tables and linear equations
- Analyzes, extends and applies visual, numeric and geometric patterns

TIME 60-120 minutes

MATERIALS

- Graph paper (included in the student lesson)
- Ruler

BACKGROUND INFORMATION
The marine environment is omnipresent in Southeast Alaska. Wherever we go in Southeast, the canals and fingers of the Pacific Ocean are part of the reality of daily life. Travel, commercial and sport fishing, and our ability to work and utilize the ocean resources at our back door are both determined and limited by wave action. Examining the wave patterns on a beach is an excellent way to apply higher math to the natural world.

Sea conditions are monitored and predicted in the daily marine weather forecast issued by the National Weather Service. The most vital information in the National Weather Service marine forecast is the predicted height of waves. If the marine forecast indicates that sea conditions in Stephens Passage are four feet, that means that half the waves will be under four feet and half the waves will be over four feet. Thus, "seas at four feet" indicates the average wave height for the specified location. The local conditions, such as a restricted channel and strong tides, can produce temporary, localized situations which result in greater wave height than the average indicated by the marine forecast for the region. For example, the area between Point Couverton and Rocky Island at the entrance to Icy Strait can generate seas conditions three feet above the marine forecast, especially when a strong incoming or outgoing tide is running against the wind.

Twenty years ago, the National Weather Service would not issue a marine forecast indicating seas less than three feet. One could be out in Stephens Passage in a flat calm, and the NWS marine forecast would indicate "seas at three feet." This was a source of amusement and some frustration to mariners in Southeast. The NWS had a policy of erring on the side of caution. In recent years, due to improved data collection, new computer software, a change in philosophy, and public understanding that local conditions can exceed the average sea condition forecast for an area, the NWS marine forecast is more specific. One can hear "seas at two feet," or even occasionally, "seas at one foot."

ALGEBRA LESSON 3 - TEACHER GUIDE INFO, CONT.

Activity 1 introduces the concepts of independent and dependent variables in a cause and effect relationship, ordered pairs, and graphing the relationship of ordered pairs. Introduce the lesson by discussing the importance of sea conditions to travel, fishing, and recreation in Southeast. If you are near a beach, you may want to have your students estimate or even measure wave height. Students who are involved in fishing will be familiar with the marine forecast. Many may have stories to share about the unique sea conditions in the area. You may wish to extend the lesson by having student monitor marine forecasts.

ACTIVITIES

Activity #1

Students describe waves rolling into a beach with algebraic expressions. The cause and effect relationship of wind speed and wave height is introduced.

Activity #2

Students graph wave data on graph paper. Dependent and independent variables are introduced.

EXPANDING THE LESSON

Take your students to a beach and estimate (or measure with a yardstick) wave height. Record and graph data for that location and time. Have students come up with the mean (average) sea conditions for that location.

Listen to the marine weather forecast on a marine radio. Marine forecasts can also be heard on PBS at specific times during the day and are published in the newspaper.

ASSESSMENT

Students accurately complete the computations in Activity 1.
Students understand the concepts of dependent and independent variable.
Students successfully plot the ordered pairs in Activity 2.
Students can write an equation describing a cause and effect relationship. (Lesson Expansion)

ALGEBRA LESSON 4

OBJECTIVES
- Learn about the salmon industry's impact on Southeast Alaska.
- Solve multi-step problems in a real-world context.
- Solving linear equations
- Solving formulas for a given variable

TIME 50 to 70 minutes

MATERIALS
- Pencil and paper

BACKGROUND INFORMATION
Commercial fishing is a staple of the Alaskan economy. Many rural families in Southeast Alaska are involved with the fishing industry, either in local waters or other areas. Commercial fishing in Southeast draws fishers and boats from all over Alaska and from as far south as California. This lesson challenges the student to determine the income and expenses incurred during a hypothetical fishing season.
Introduce this activity to your students by asking if any members of the class has worked aboard a commercial fishing boat or has a family member working in commercial fishing. Ask these students to tell about their family connection with commercial fishing. Take your students into the mindset of a captain aboard his boat–his concerns and his aspirations for the season. Include revenue, expenses, and profit in the discussion. What does it take to make a successful season?

MATH CONCEPTS
Viewing an = sign as the balance of a scale.
The variable represents something with units attached.

KEY POINTS
Protect the balance of the scale.
Whatever you do to one side of the scale (= sign), you must do the same to the other side of the scale.

ACTIVITIES
Activity 1: Students find the average weight of one fish using simple algebraic expressions.
Activity 2: Students calculate the anticipated expenses for a fishing boat for one season.
Activity 3: In this activity, the students are asked to calculate the average catch per boat during a 12 hour opener.
Activity 4: This activity involves three distance, rate and time problems
Activity 5: Students determine average fuel consumption per day and determine if the fuel consumption is consistent with the estimated fuel expense.
Activity 6: Students calculate the expenses for the season and determine if the season was profitable, based on a fixed price for a catch of 94,650 pounds of sockeye salmon.

ALGEBRA LESSON 5

OBJECTIVES

- Learn to calculate flow rate and ice melting rate to determine the amount of movement of the glacier.
- Involve the properties of exponents in the calculations of movement
- Calculate volume of the glacial ice.
- Adding and multiplying polynomials
- Simplifying and evaluating expressions
- Expressing values in scientific notation

TIME 60-120 minutes

MATERIALS

- Pencil and paper
- Calculator

ALASKA MATHEMATICS STANDARDS (adopted June 2012)

- Geometric Measurement and Dimension (G-GMD), Explain volume formulas and use them to solve problems
- Modeling with Geometry (G-MG), Apply geometric concepts in modeling situations

ESSENTIAL SKILLS

- Multiplying and dividing monomials
- Properties of exponents
- Scientific notation
- Add, subtract, and multiply polynomials
- Special products

THEORY

Properties of exponents – highlight the correlation between multiplication and addition, and division and subtraction.

Zero exponent – by definition, not intuition

Apply these properties of exponents to scientific notation for expressing very large and very small quantities.

ALGEBRA LESSON 5 - TEACHER GUIDE INFO, CONT.

ACTIVITIES

Activity 1: This activity is based on the Mendenhall Glacier near Juneau, which is receding at a rapid rate. Students will calculate annual rate of recession of the glacier by subtracting the total ice loss from the flow rate at the terminus.

Activity 2: In this activity students are given the dimensions of an iceberg visible above the surface and will calculate an algebraic expression for estimating the total volume.

Activity 3: Given a quantity of icebergs, students will create an expression for the total volume of ice breaking off over a period of time.

ASSESSMENT

Students successfully calculate the rate of ice loss in Activity 1.
Students calculate the amount of time it takes for the glacier to travel a certain distance in Activity 1.
Students calculate how quickly (at what rate) ice is breaking off the glacier and causing it to recede in Activity 1.
Students will successfully calculate the volume of an iceberg and express this value in algebraic terms in Activity 2.
Students will calculate the average and total volume of ice breaking off the glacier during a specific time period in Activity 3.

ADDITIONAL NFORMATION

These activities make use of scientific notation. Review the basics of scientific notation as you introduce the activities to your students. Scientific notation is a way of writing numbers which are too big or too small to be written in standard decimal form. Numbers are written in the following form: (a * 10^b = a times ten raised to the power of b) Thus 3 can be written as 3 * 10^0

300 can be written as 3 * 10^2
and
4444 becomes 4.444 * 10^3

OPTIONAL EXPANSION ACTIVITIES

Repeat this activity using the movements of the Herbert Glacier as a model (instead of the Mendenhall Glacier). Apply this activity to the glacier nearest you, contacting the local Forest Service if further information is necessary to complete the objective.

GEOMETRY LESSON 1 - MEASURING HEIGHT

OBJECTIVES

- Learn to determine a baseline (the distance from your position to the base of the object you want to measure).
- Learn to measure angles for large, fixed objects, such as a building.
- Learn to convert measurements from standard measurement (inches, feet, etc.) into fractions.

TIME 120-180 minutes

MATERIALS

- Standard tape measure
- Standard protractor,
- String
- Eraser
- Soda straw
- Tangent conversion table
- Calculator with a tangent function

BACKGROUND INFORMATION

Alaskans are accustomed to living in a mountainous environment. The vertical dimensions of our geography orient us as we move about Southeast whether we're on a boat, aircraft or our limited road system. We orient ourselves by recognizing ridges, mountain peaks, glaciers, islands, inlets and bays.

Our consciousness adapts to the vertical dimensionality of the landscape. When transplanted to a flat geographic environment, the brain of a long time Southeastern resident rebels against the featurelessness of the terrain. The lack of vertical landmarks registers in the brain as vague sense of disorder. Fishing off the eastern seaboard or the Gulf Coast of the United States is a strange experience for those of us accustomed to the bays and channels of Southeast. The flat coastline appears featureless and boring to Alaskan eyes.

This geometry lesson addresses our fascination with heights. Students are challenged to apply math skills to determine the height of features in their environment, such as a: flag pole, building, tree or mountain. Introduce the lesson with a discussion of the vertical features in the immediate landscape. Have students read the account of Indian mathematician, Radhanath Sikdar measuring the height of Mount Everest from a distance of 150 miles.

Students can build their own angle measuring device for this lesson. Materials needed to construct this device include a protractor, string, soda straw, and an eraser (or other materials to serve as a weight, such as a nut, small bolt, etc.) A more sophisticated (and accurate) angle measuring device can be ordered from Estes Model Rocket Company at the following web address: http://www.estesrockets.com/

GEOMETRY LESSON 1 - TEACHER GUIDE INFO, CONT.

The lesson includes a presentation of sine, cosine and tangent in Expanding the Lesson. Exercise answers will vary in this lesson depending on features being measured. In order to check for accuracy, have the students work in pairs and have more than one pair of students measure the same object.

ESSENTIAL SKILLS

Measuring angles
Determining elevation
Azimuth
Sine and Tangent
Creating an angle sighting device

CONCEPTS

The height of natural and man-made objects can be calculated using triangles.
The ratio of the height of a triangle to the hypotenuse is called the sine.
The ratio of the base of a triangle to the hypotenuse is called the cosine.

Alaska Mathematics Standards (adopted June 2012)
Geometric Measurement and Dimension (G-GMD), Explain volume formulas and use them to solve problems

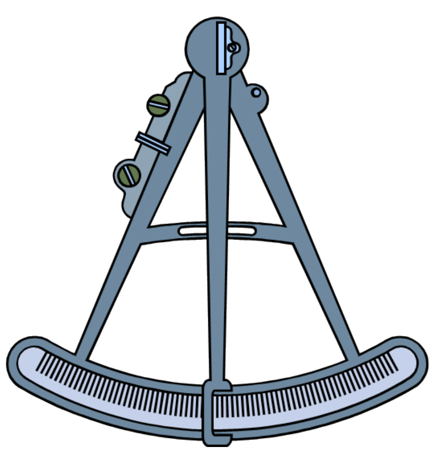
Define trigonometric ratios and solve problems involving right triangles.

G-SRT.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G-SRT.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

ACTIVITIES

Students construct a homemade "sextant" with a compass, string and eraser prior to beginning the activities in the lesson.



GEOMETRY LESSON 1 - TEACHER GUIDE INFO, CONT.

Activity 1: Using the homemade "sextant," students measure the angles by sighting the highest point of five different features (i.e. buildings, a flagpole, trees, mountains, etc.) Students can measure the baseline to a nearby object and determine the height by multiplying the baseline by the tangent of the angle (see the Tangent Table in the student lesson). This activity also includes a definition and explanation of tangent. This activity is best completed in pairs.

Activity 2: Activity 2 provides more practice as the students work in pairs to determine the height of a near-by building.

Activity 3: Teams of students measure the height of four natural or man-made features.

ASSESSMENT

Students accurately measure angles in Activity 1.
Students accurately measure the height of a nearby building in Activity 2.
Students successfully measure the height of four man-made or natural features.
Students accurately interpret and apply conversions of tangent, sine and cosine.

OPTIONAL EXPANSION ACTIVITIES

See the lesson on page 9 for explanations of sine, cosine, tangent and mathematical functions.

Algebra Lesson 1

Tlingit Phrase: Yáadu X̲áat!

English Translation: The fish are here!

Lesson 1 - Linear Equations

The earth revolves around the sun in an annual cycle, creating the seasons. The Northern Hemisphere shifts through the annual cycle receiving the sun's energy in varying amounts. Alaska receives the most solar energy when the earth's axis is tilted toward the sun. This takes place during the months of May, June, July and August.

Summer in Alaska, when the days are long and the ground warms from greater periods of exposure to the sun's heat, is when all living things become more active. Seasons unfold and life cycles in Alaska become intertwined. Salmon come home, driven by an undeniable desire to return to the very stream in which they were born. They return by the millions. Bears and eagles congregate to the streams and we humans break out our nets and fishing rods in pursuit of this precious food source.

All life--plants, and animals--are connected. The world is such a complicated place that it is beyond our capability to precisely describe all the connections or every action/reaction. There are just too many **variables**. This is where mathematics can help us. If we don't know what a particular quantity is, we can assign the unknown quantity a name, such as X. Once we assign a variable (X) to represent an unknown quantity, then we can fit the unknown in to its rightful place in an equation.

In the following example, we do not know the weight of each and every fish that John caught, but we can call the weight of a single fish X and then place X into an equation which describes what we do know about the fish. Let's examine several situations where we have an unknown number. We will call this unknown number **X**.

SOLVING LINEAR EQUATIONS

Yesterday John caught a fish. He stepped onto a scale while holding his fish and the scale read 192 pounds. If we know that John's weight is 175 pounds, how much did his fish weigh?

17 Pounds
(answer)

You can easily answer this question. Intuitively you might think that if 192 lbs. is the total weight, and 175 lbs. is John's weight, then subtracting his weight from the total will give the weight of the fish: 192 lbs. – 175 lbs. = 17 lbs. The weight of the fish is 17 pounds!

Let's look at the problem another way. The total weight is John's weight plus the weight of the fish. We don't know the weight of the fish, so let's call it X. This situation can be written as an equation:

The fish's weight plus John's weight is the total weight.

$$X + 175 \text{ lbs.} = 192 \text{ lbs.}$$

We take away 175 lbs. from each side to solve for X = 17 lbs.

$$X + 175 \text{ lbs.} - 175 \text{ lbs.} = 192 \text{ lbs.} - 175 \text{ lbs.}$$
$$X = 17 \text{ lbs.}$$

The idea is to get X by itself. The kicker is that whatever we do to accomplish this must be done on both sides of the equal sign.

Activity 1

This time John caught two fish. The second fish is five pounds heavier than the first. When John steps on the scale holding his two fish, the scale reads 197 pounds. Can you find the weight of each fish? Give it a try:

8.5 AND 13.5 LBS

(answer)

How did you do? Maybe your intuition carried you through. Maybe you used algebra. Maybe the two are not so different.

Let's try an algebraic approach to solving this problem.

We don't know the weight of either fish, so we start by assigning a variable to the weight of one of the fish:

Let X = weight of the first fish

Then we know $X + 5$ = weight of the second fish

Here's what we know:

The weight of the first fish plus the weight of the second fish plus John's weight equals the total weight.

$$X + (X + 5) + 175 = 197$$

Let's solve this equation together. Combine the terms on the left side of the equation. Our equation now looks like this:

$$2X + 180 = 197$$

Then subtract 180 from both sides of the equation.

$$2X + 180 - 180 = 197 - 180$$
$$2X = 17$$

Now, divide both sides by 2. $X = 8.5$ lbs. This is the weight of John's first fish.

$X + 5 = 13.5$ lbs.   This is the weight of John's second fish.

Activity 2

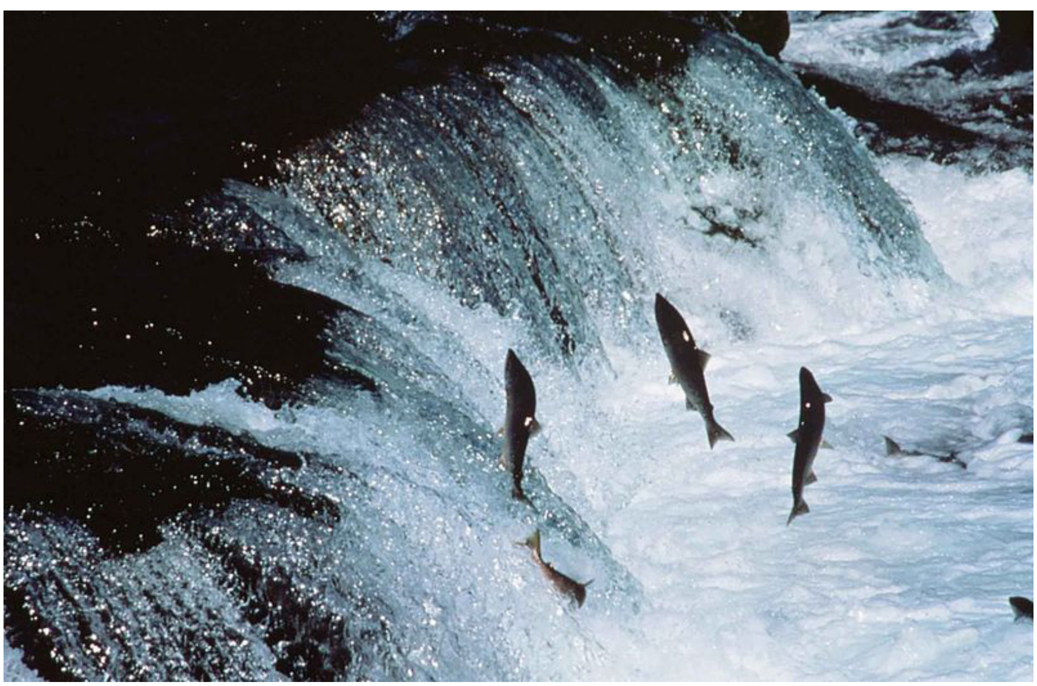
James and Tim have set up camp near a small stream on Admiralty Island. They have stowed their gear carefully, with no food or fishing tackle near the camp. Coho are running up stream at a rate of 8 fish per hour. Tim estimates that the Coho vary in weight from 8 to 12 pounds. Assuming that the average Coho is 10 pounds and that the run continues at the same hourly rate, what will be the total biomass *(that is, the weight of all the returning Coho)* added to the headwaters of the stream during a ten day period? *(Hint--let X be the weight of the biomass.)*

X = biomass
8 Coho per hour
10 lbs. average weight per fish
X = (8 x 10lbs.) multiplied by (24 hrs x 10 days)
X = 80 lbs. multiplied by 240 hours
X = 19,200 lbs.

Total weight (*biomass*) of returning Coho in ten days is 19,200 LBS

The whole ecosystem, including bears, eagles, and even hungry halibut downstream feed directly off the dead Cohos. Even a modest salmon run in a small stream adds a large amount of nutrients to the forest ecosystem. All life in the rainforest benefits from this yearly infusion of nutrients.

Did you know that the salmon eggs will not hatch if the water in the stream becomes too warm? Overhanging branches from trees are essential to shade the stream and maintain the cool environment needed by the eggs. Two weeks of uninterrupted sunshine in August may please the tourists, but the effect on salmon eggs can be disastrous.



Activity 3

The female Coho salmon digs a nest with her tail and deposits from 2,400 to 4,500 eggs. Assuming that half of the returning salmon are female, how many eggs will be deposited in the stream during that ten day period of time? (*Hint—use the data from Exercise 2. First determine the number of Cohos returning during the ten day period and divide by two. Assume that the average female deposits 3,450 eggs. Let X be the total number of eggs.*)

Number of returning salmon: X = 8 x 24 x 10 days
X = 8 x 240
X = 1920 Coho returning in 10 days
1920 divided by 2 = 960 female

X = 960 X 3,450
X = 3,312,000 eggs

Number of eggs deposited in ten days: 3,312,000 EGGS

Salmon Eggs

Activity 4

Several days after camping near the stream, James learned that two years previously an Alaska Department of Fish and Game biologist had conducted a study of this stream. According to the biologist, bears are particularly active near this stream during the ten day Coho run. Predation of the spawning Cohos from bears is estimated to be four percent of the run. In that same ten day period, assuming the run is steady, how may Cohos *(both male and female)* will be taken by bears? How many eggs will be lost to bears?

X = 1920 x .04
X = 76.8 Coho

Number of Coho taken by bears: 77 Coho (rounded-up)

Let us assume that half the returning Cohos are female.
77 diided by 2 is 38.5. We will round this number up to 39 to give us 39 female (egg-bearing) Cohos lost to bears.

Y = 3,450 eggs multiplied by 39 female Cohos
Y = 134,550 Eggs

Number of eggs lost to bears: 134,550 eggs

Algebra Lesson 1 Extension

Locate a nearby salmon stream. Ask parents, grandparents, and Elders about the history of the salmon runs in the stream. Have the runs changed over time?

Do an actual stream study. Count the number of each species of salmon passing a given point in a half hour period for a specific number of days--five to ten days. If you can, sample several fish and determine an average weight. Determine the duration of the salmon run. Based on the data which you have gathered, calculate the biomass of the salmon run and the number of salmon eggs deposited in the stream. If you have an active bear population, you can also factor predation into this calculation.

Sockeye Salmon Spawn

Algebra Lesson 2

Tlingit Phrase: Goosóo wé aas gutóode?

English Translation: Where is the forest?

Lesson 2 - The Amazing Life of Trees

Southeast Alaska is the home of the Tongass National Forest - the largest area of temperate rain forest on the planet. If you have ever hiked through a lush forest, you may have been struck by a sense of timelessness; a pervading calm that removes you from every day life. The forest is a world at peace, where change and growth happens over years, decades, and centuries. In our oldest forests in Southeast Alaska, trees may live for more than 800 years, gain a diameter of 12 feet, and grow to a height of 200 feet or more! Imagine the life of such a tree.

- During the tree's lifetime, the forest beds have been thick with vegetation, tightly fixing the ground with intermingling root systems.
- Local deer will have sought out the shelter of the tree from storms and fed on the undergrowth pro tected by its canopy.
- Returning salmon have found ideal spawning grounds in streams near this tree.
- The temperature of the stream is regulated by the shade of this great tree and its neighbors.
- The stream's current is broken by branches and fallen trees, making a perfect refuge for laying eggs.
- The salmon in streams near this ancient tree have flourished because of these conditions, thus adding to the health of oceans.
- Life gives life, as bear, eagles, and other birds feed on the salmon.

During this tree's existence, life abounded in the region. Local **Tlingit**, **Haida**, and **Tsimshian** tribes have enjoyed abundance. In the last 100 years, many of its neighboring trees have fallen to a logger's blade and a growing timber industry. Left to natural cycles, the forest weaves an intricate balance to sustain a complex ecosystem, allowing the diversified animal and plant life of Southeast to prosper. In a world where time is measured on a greater scale, day to day events become diluted in a much larger pool of time. Giants of our world, the old growth trees are elders to us all. With them, we can find solace. With them we can find the wisdom that comes with age.

In the last fifty years, much of the old growth forest of Southeast Alaska has been logged. Major portions of the Tongass Rainforest have experienced a reduction in old growth forest and the ability of the land to support life. As the large old growth trees are harvested, the rich ecosystem they support deteriorates. The land becomes less able to support deer, wolves, eagles, and bear.

HOW OLD ARE OUR TREES?

The best way to determine a tree's age is to count the annual rings. Each ring represents one year of growth. The lighter and wider part of each ring grows during spring and summer, the plentiful time of year, when growth is fast. The darker part of each ring is added during fall and winter, when nutrients are not as plentiful and growth is slower.

Each ring, from light to dark, represents the cycle of seasons for one full year. As you can see in the picture, each year's ring varies in width. The innermost rings are from when the tree was young and growing very fast. The outer rings are more recent when the tree has aged and its outward growth has slowed considerably.

The rings of a tree provide us with a snapshot into the past, a history of climate conditions over the life of the tree. Ring widths depend on many factors, like the species of the tree, how much competition the tree has for sunlight, regional climate, annual rainfall, etc. The weather conditions during a particular year are reflected in the width of tree ring. Each species of tree will have an average ring width. Foresters use this information to determine a tree's approximate age by measuring the tree's **diameter**.

Activity 1

The forested area off the end of Thane Road in Juneau has been logged. If you walk through this area, you would notice that most of the trees are similar in size. This is because they are all about the same age.

You are investigating the age of a stand of trees on Thane Road. In order to get an *average* age, you decide to measure the circumference of 10 trees. The **circumference** of a tree is the distance around it, and you decide this would be the easiest measurement to take. Your tape measure only measures in feet and tenths of a foot. These are the measurements you have taken:

Table 1

Tree #	1	2	3	4	5	6	7	8	9	10
Circumference (in feet)	4 8/10	5 3/10	4 2/10	5 1/10	4 3/10	5 5/10	4 7/10	5 6/10	5 2/10	4 9/10

Since 1 foot = 12 inches, we can change from feet to inches by multiplying by 12.
For tree #1 this gives us:

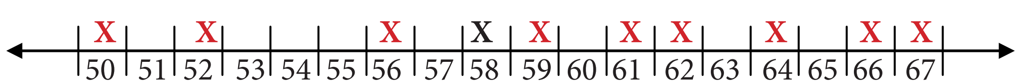
$$4\frac{8}{10}x12 = \frac{48}{10}\cdot\frac{12}{1} = \frac{48}{5}\cdot\frac{6}{1} = \frac{288}{5} = 57\frac{3}{5}in$$

For trees #2 — 10, change from feet to inches. Keep your work in fraction form and enter the answers in **table 2**.

Table 2

Tree #	1	2	3	4	5	6	7	8	9	10
Circumference (in inches)	57 3/5	63 3/5	50 2/5	61 1/5	51 3/5	66	56 2/5	67 1/5	62 2/5	58 4/5

One way to display the measurements in table 2 is to make a **line plot**. Round each of the measurements in table 2 to the nearest inch and put an X above the appropriate number on the line below. The measurement for tree #1 has been done for you.



Now that we have the Circumference, C, for each of our ten trees, we can use these measurements to find the Diameter, D, for each tree. Tree diameter is measured at chest height, or about 4.5 feet above ground level.

The Circumference of a circle is the product of the circles Diameter and pi (∏). Multiply the circle's Diameter by ∏ to determine the Circumference. Translate this last sentence into a formula involving C and D and write the formula on the line below.

$C = \pi D$

(answer)

Another way of stating the last formula is to say that the Diameter, D, of a circle is the ratio of its Circumference, C, and pi. This formula can be written as follows:

$$D = \frac{C}{\pi}$$

Since we now have the Circumference for each tree in **Table 2**, we can use this formula, and the fact that ∏ is approximately 3.14 (∏ ≈ 3.14), to calculate the Diameter of each of our ten trees.
Use the information from Table 2, and the formula $D = \frac{C}{3.14}$, to calculate the Diameter of each tree.

You should change each measurement to a decimal first, then use your calculator. Round each diameter to the nearest tenth of an inch.

For tree #1 this gives: $57\frac{3}{5} = 57.6$ in.

The Diameter is $D = \frac{57.6}{3.14}$ = 18.34394904 inches. .

If we round this off to the nearest tenth of an inch, we get D = 18.3 inches.

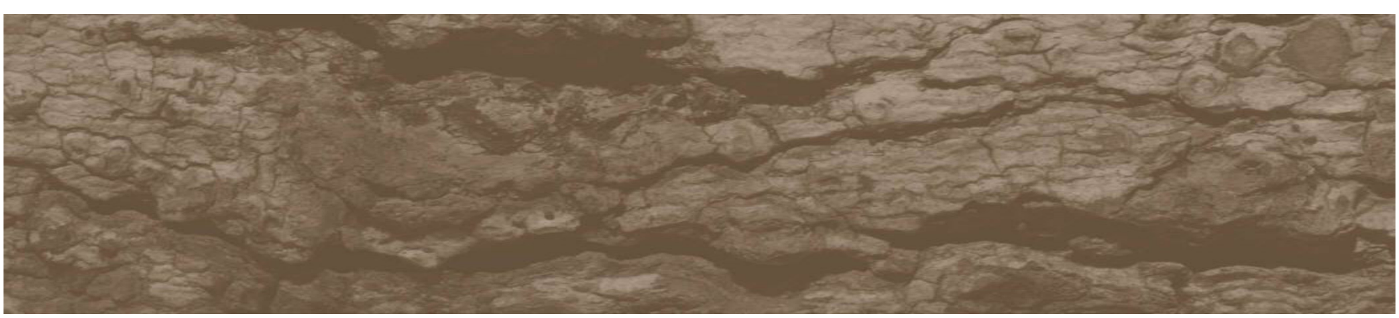
Enter the Diameters of trees #2 — 10 in **Table 3**.

Table 3

Tree #	1	2	3	4	5	6	7	8	9	10
Diameter (in inches)	18.3	20.3	16	19.5	16.4	21	18	21.4	19.9	18.7

Use the measurements in Table 3 to calculate the *average diameter* of the ten trees:

Average Diameter = $\frac{189.5}{10}$ = 18.95 inches



CALCULATING BOARD FEET IN A STANDING TREE

The volume of lumber is calculated in board feet. A board foot is an amount of wood measuring one foot by one foot by one inch. In this activity, let's assume that we have already measured the diameter of ten trees. Since these trees are being measured for logging, the height of these ten trees is measured up to the minimum merchantable log diameter, where the tree is approximately 8 top 10 inches wide at the top. These measurements are recorded in **Table 4**.

Table 4

Tree #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Height **(in feet)**	45	52	57	60	55	49	61	55	59	57

With this information, we can perform the following steps in order to determine the board feet available in each standing tree.

1. Calculate the Area of the surface of a theoretical slice of the tree from the diameter measured at chest height. (see Table 3)

Area = 3.14 multiplied by the radius squared
(or Area = (diameter/2)2 x 3.14 square inches)

Example: Tree #1 has a diameter of 18.3 inches. The radius is 9.15 inches. Applying the formula:

9.15 squared is 83.7
83.7 multiplied by pi (3.14) = **262.9 square inches**

2. Convert the area in square inches to square feet by multiplying as follows:

Area (*in square inches*) x .00694 = Area (*in square feet*)

Tree #1: 262.9 square inches multiplied by .00694 = **1.8 square feet**

3. You can now calculate the potential Cubic Feet of lumber in the tree by using the following formula:

Cubic Feet of Lumber in the Tree = Area (*square ft.*) x Height (*ft.*)/4
(This formula was developed by the wood processing industry to determine the cubic feet of wood from a mathematically idealized tree.)

Tree #1: 1.8 square feet multiplied by 45 (*the height of the tree from Table 4*) = 82
Now, divide 82 by 4 and you get **20.5 cubic feet of lumber**.

The last step is to convert cubic feet of lumber to board feet.

Board Feet = Cubic Feet x 12 (*Cubic Feet multiplied by 12*)

Tree #1: 20.5 cubic feet of lumber multiplied by 12 = **246 board feet of lumber**

This value is added in **Table 5**. Complete Table 5 by doing these calculations for Trees 2 through 10. Record the number of potential board feet of lumber from each tree in **Table 5**.

Table 5

Tree #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Board Feet	246	350	239	373	241	353	323	412	382	302

Using the values in **Table 5**, calculate the average board feet of the trees, rounding to the nearest board foot. (*add the total board feet and divide by the number of trees.*)

3221 divided by 10 = 322 Board Feet

Average Board Feet = 322 BOARD FEET

CHALLENGE ACTIVITIES:

Activity #1: Use this method to calculate the board feet of lumber in a tree of known height with a top taper of 8 to 10 inches. (*See **Geometry Lesson 1** for a method of determining the height of a tree mathematically.*) Or if you like, use this method to calculate the board feet in a log lying on the ground.

Height of the Trees: ___________________

Area of Surface: ___________________

Square Feet of Surface: _____________________

Cubic Feet: ____________________

Board Feet: ____________________

Activity #2: Select a specific tree. Calculate the board feet as in **Activity 1**. Then determine the specie of the tree. Do research on the internet to determine the current value of a board foot of wood for that specie of tree. Then calculate the current dollar value of the wood in the tree.

Activity #3: Calculate the economic value of the wood in a stand of trees.

Algebra Lesson 3

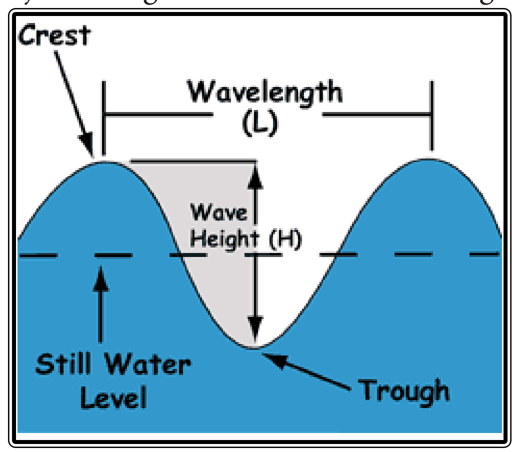
Tlingit Phrase: Óoxjaa tóox̲ yaa kak̲úx.

English Translation: It (boat) is travelling in a wind.

Lesson 3 - Measuring Wavelength

The ocean is part of the natural world around us. It's part of our daily lives, even though we may not always notice it. Have you ever been riding to school, looked out into the channel and thought, "Wow, it's really rough out there today," or "Man, it's glassy calm"? These are typical ways that we might describe the ocean.

If you are on the phone to a friend in Dutch Harbor, and you say that the ocean is rough today, you have an image in your mind of what you mean. Your friend might interpret your statement with an entirely different image. Maybe your friend is a fisher in the Bering Sea, so rough conjures images of fierce waves breaking over the bow of the boat. The same description – rough – might be interpreted by a person living on a lake in California as two foot chop, with occasional white spray blowing off the crests of the waves.



Descriptions like these are **relative** to the person's experiences. They are flawed from the outset because they mean different things to different people. We call descriptions like these **qualitative**, because they describe the qualities a situation has (*big, small, rough, calm, etc.*)

We can convey our meaning more clearly if we quantify the waves we are describing. A **quantitative** description involves assigning numerical quantities to what we are describing. The height of a wave is measured from the trough in front of it to its crest. (*See Illustration*) While, on any given day, waves will have varying heights, the average wave height is a good description of how rough the sea conditions are. To say, "The waves are three and a half feet today," is a description that will mean exactly the same thing to the fisher in the Bering Sea as it will to the person beside the lake in California. In this way mathematical descriptions of our world translate across language and experience barriers.

Another way we can accurately describe a wave is to compare it to nearby waves. "That wave is twice as big as the one before it," would paint a good picture, or "It seems like every seventh wave is about two feet higher than the others." If we know the size of the wave we are comparing to, a picture of the surrounding waves would come into focus. This way, we can express each succeeding wave relative to the size of the first wave.

Activity 1

Let's try to write a quantitative description of the waves rolling in at a beach in Southeast Alaska. While on the beach, you begin to notice a pattern in the waves crashing on shore. For each wave, write an algebraic expression for its size in terms of the first wave. We will say that the first wave's height is x feet.

Wave # and Description	Algebraic Expression
1. x feet high	x
2. 1 foot more than the first	$x + 1$
3. 1 foot less than the first	$x - 1$
4. The third wave, decreased by 1 foot	$x - 2$
5. Twice as big as the fourth wave	$2(x - 2)$
6. Three feet more than the fourth	$x - 2 + 3$ or $x + 1$
7. 1 foot more than twice the fourth	$2(x-2) + 1$
8. 2 less than the sum of the last two waves	$(x + 1) + 2(x - 2) + 1 - 2$

To find the **average** wave height, we add all the heights together and then divide by the number of waves. On the space below, write an algebraic expression for the average wave height. Use the algebraic expressions you wrote above to find the expression for the average wave height.

Avg Wave Height: $\frac{x + (x + 1) + (x - 1) + (x - 2) + 2(x - 2) + (x + 1) + 2(x - 2) + 1 + (x + 1) + 2(x - 2) + 1 - 2}{8}$

Simplify the numerator by combining like terms. The result is a formula for determining the Average Wave Height.

Avg Wave Height: $\frac{12x - 12}{8} = 1.5x - 1.5$

(Circle the Average Wave Height formula. You will use it soon.)

The wind is now blowing at 20 mph and the height of the first wave is 3 feet. List the heights of the eight waves using your eight algebraic expressions with x = 3.

1. 3 ft.	5. 2 ft.
2. 4 ft.	6. 4 ft.
3. 2 ft.	7. 3 ft.
4. 1 ft.	8. 5 ft.

Calculate the Average Wave Height by using simple arithmetic. That is, add each of the wave heights and divide the sum by 8.

$3 + 4 + 2 + 1 + 2 + 4 + 3 + 5 = 24$

$24/8 = 3$ ft

3 FT
Average Wave Height

Now, use the algebraic formula to find the Average Wave Height. (Remember, x = 3)

Answer: $1.5x - 1.5 = 1.5(3) - 1.5 = 4 - 1.5 = 3$ ft

3 FT
Average Wave Height

Do your answers agree? Yes X No ____

The wind picks up from 20 mph to 30 mph. You notice that the same pattern persists, but all the waves have increased in size. Now the first wave is 4 feet high. (*x = 4*)

List the sizes of the eight waves.

1. 4 ft.
2. 5 ft.
3. 3 ft.
4. 2 ft.
5. 4 ft.
6. 5 ft.
7. 5 ft.
8. 8 ft.

Calculate the Average Wave Height arithmetically using your list.
(*Add all eight wave heights and divide by 8.*)

$4 + 5 + 3 + 2 + 4 + 5 + 5 + 8 = 36$

$36/8 = 4.5$ ft

4.5 FT
Average Wave Height

Now, calculate the Average Wave Height with algebra using the Average Wave Height formula. (x = 4)
Do your answers agree? Yes X No ____

How does this answer compare with your first average?

Answer: 1.5 FEET HIGHER THAN THE FIRST AVERAGE

If the wind increased from 30 mph to 40 mph, what do you suppose the average wave height would be?

Answer: AVERAGE WAVE HEIGHT = 6 FT

Let's check to see if you are correct. When the wind blows 40 mph, the size of the first wave is 5 feet. Use any method to calculate the Average Wave Height.

6 FT

Average Wave Height

Write a statement about the relationship between wind speed and the average wave height at the beach. Try to give a quantitative description.

Average wave height increases by 1.5 feet for every increase in 10 mph increase in wind speed. For example: a wind speed of 50 mph would result in an average wave height of 7.5 feet and a 60 mph wind would create an average wave height of 9 feet.

Noticing patterns is the key to understanding the world around us. Mathematics allows us to describe nature in terms that give everyone the same picture.

In the last activity, we identified a relationship between the size of the waves and the speed of the wind. The size of the waves *depends* on the speed of the wind. This is a **cause and effect** relationship. Wind is the cause, and waves are the effect. The reverse would not be true. The speed of the wind does not depend on the size of the waves.

In this relationship, wind speed is an **independent variable** and wave size is the **dependent variable**. Another way of stating this *dependent-independent relationship* is to say "the size of the waves is a *function* of the speed of the wind." The expression "is a function of" means "depends on."

Other examples of dependent-independent relationships are: "The number of *pounds* of turkey you eat at Thanksgiving is a function of how hungry you are," and "The distance *you* travel on your bicycle in 30 minutes is a function of how fast you peddle." Describing a relationship quantitatively will be our goal in the next exercise.

Activity 2

At the end of Activity 1 you described the relationship between wind speed and the average wave height at a beach. One possible description might be, "*As the wind speed increases, the waves get bigger.*" A more precise description might be, "*Every time the wind increases by 10 mph, the size of the waves increase by 1.5 feet.*"

We could describe this relationship even more precisely by using an equation: H = 0.15w, where H = the height of the waves in feet, and w = the wind speed in mph.

- When w = 20, H = 3 exactly as we saw in the last exercise.
- Also, when w = 30, then we can find H by substituting 30 into the equation: H = 0.15(30) = 4.5. This also agrees with what we saw at the beach.
- Each of these can be considered as a w-input, along with an H-output, and can be written as an **or dered pair**, (w, H). The ordered pairs would be (20, 3), and (30, 4.5).
- **Important point**--in an *ordered pair* like (w, H), the first coordinate is the **independent variable**, and the second coordinate is the **dependent variable**.

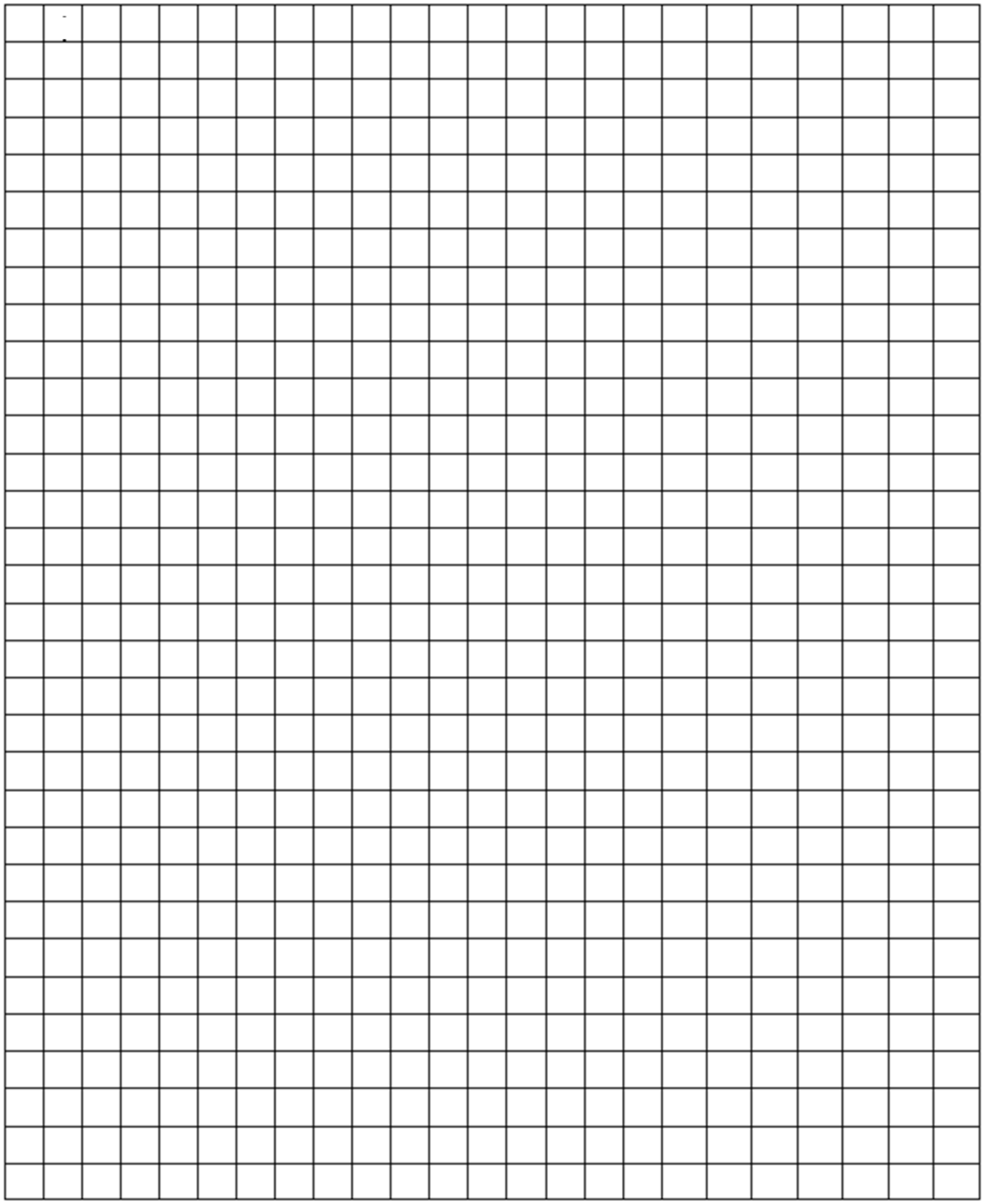
Use the equation H = 0.15w to fill out the table of values and find some ordered pairs:

w input	H output	(w, H) ordered pair
0	0	(0, 0)
10	1.5	(10, 1.5)
20	3	(20, 3)
30	4.5	(30. 4.5)
40	6	(40, 6.0)
50	7.5	(50, 7.5)

Each of these ordered pairs can then be plotted on a graph to show the height of the waves, H, as a function of wind speed, w. Graph the rest of the ordered pairs and draw the graph:

On a graph, the horizontal coordinate is always the independent variable (*w in this case*) and the vertical coordinate is always the dependent variable (*H in this case*). Now, graph these ordered pairs on graph paper. On your graph, extend the order pairs out to include a wind speed of 80 mph.

(DEPENDENT VARIABLE)



INDEPENDENT VARIABLE

Algebra Lesson 3 Extension

Cause and effect relationships surround us every day. Make three statements showing a dependent-independent relationship. You can use your imagination with this, but keep in mind that you must try to quantify one of these relationships:

1. "____________________________is a function of ________________________________"

2. "____________________________is a function of ________________________________"

3. "____________________________is a function of ________________________________"

Choose one of your ordered pairs. Make a table of values to get several ordered pairs.

Input	**Output**	**Ordered Pair**

What is the independent variable? ______________________________________

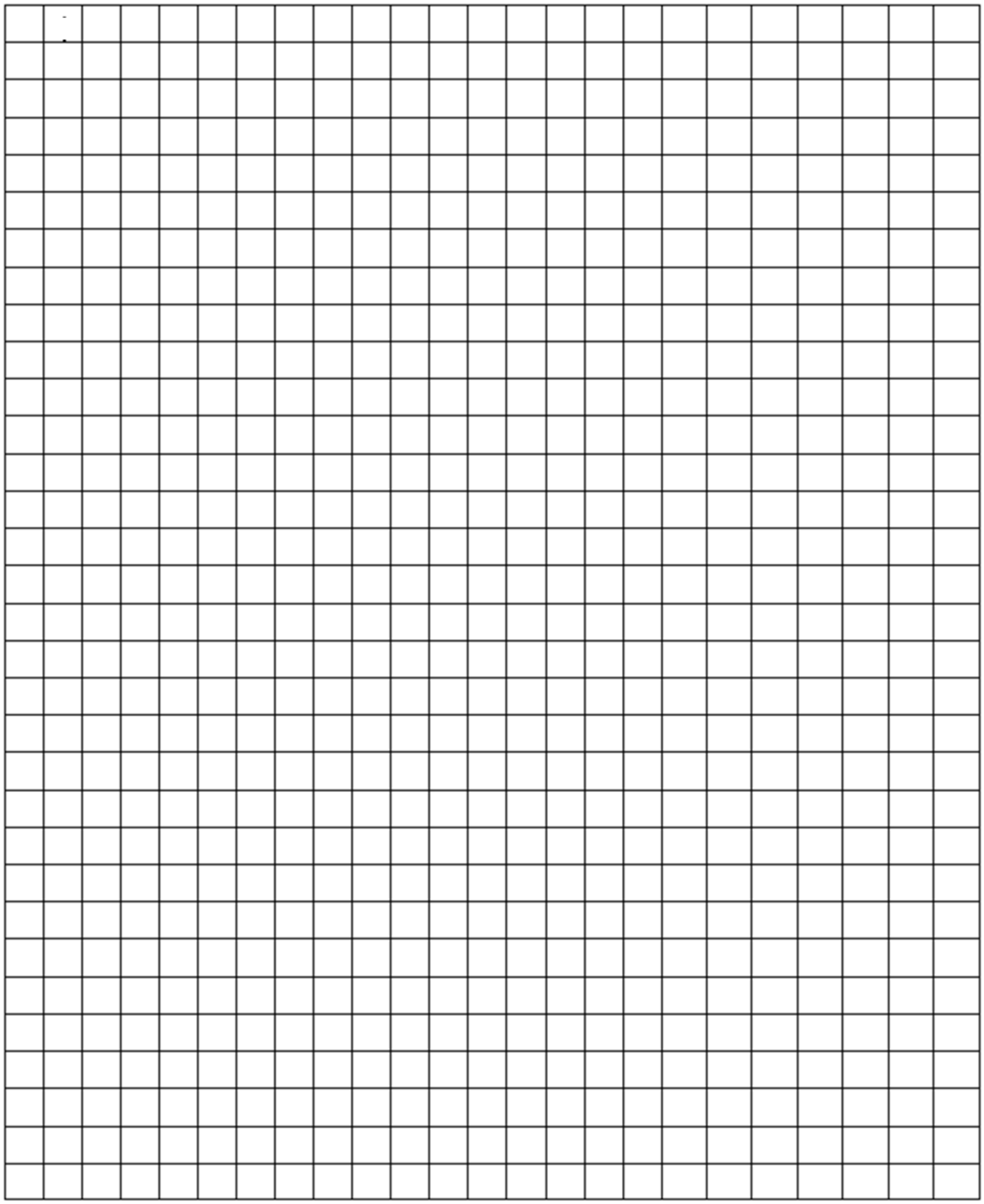
What is the dependent variable? ______________________________________

Write an equation describing the relationship:

____

Now, graph the relationship on the graph paper. Label the graph correctly.

(DEPENDENT VARIABLE)



INDEPENDENT VARIABLE

Algebra Lesson 4

Tlingit Phrase: Shukalx̱aach.

English Translation: S/he's trolling.

Lesson 4 - Commercial Fishing Business Simulation

Imagine that you are the owner and captain of a fishing boat. In addition to being an experienced fisher, you must be a competent business person. Each fishing season you must meet your annual business expenses. These expenses begin to accumulate even before the fishing season begins. Pre-season expenses may include the boat payments, cost of travel, insurance, groceries, and boat maintenance. During the fishing season, major expenses include fuel, pay for crew, groceries, and unexpected maintenance. And every day you hope that your skill, experience, and good luck allow you to catch enough salmon to pay your expenses and make a profit. Let's do several exercises involving the business of salmon fishing.

Activity 1

Let x = average weight of one fish.

On the first set:

4 fish of average weight	$4x$
2 fish that are 1 lb. less than average	$2(x-1)$
3 fish that are 2 lbs. over the average	$3(x+2)$

The total weight is 49 lbs.

Solve $4x + 2(x-1) + 3(x+2) = 49$

What is the average weight of one fish?

$9x + 4 = 49$

$9x = 45$

$x = 5$ lbs

Answer: 5 LBS

Activity 2

Here are the anticipated **expenses** for the fishing season:

Boat Payment	$9,000
Fuel	$4,500
Maintenance	$8,000
Food	$1,600
Crew Pay	$32,000
Insurance	$2,300
Travel	$2,100
Miscellaneous	$3,500

Calculate the total expenses.

9,000 + 4,500 + 8,000 + 1,600 + 32,000 + 2,300 + 2,100 + 3,500 = 63,000.00

Total Expenses = $63,000.00

Now, let's calculate total revenue we will need to break even—that is, to pay our **expenses** for the season. Let x = the number of lbs. of fish caught. We are going to fish exclusively for sockeye this season. Based on our experience over the last several years, we expect the price of sockeye to average $1.40 per pound this season. Given this price per pound, find the number of pounds of sockeye you will need to catch to break even.

x = $63,000 divided by $1.40

x = 45,000 lbs of sockeye

Activity 3

During a single 12-hour opener, a total of 231,000 fish were caught by two hundred twenty-seven fishing boats. An average fish weighed 6.3 pounds. Find the average catch per boat by weight and price.

Average weight of catch per boat:
x = 1,455,300 lbs divided by 227 boats

x = 6,411 lbs. per boat

Average value of catch per boat:
x = 6,411 lbs. x $1.40 divided by 227 boats
x = $1,062,369 divided by 227 boats

x = $8,975.42 per boat

We caught 1238 fish. Did we miss, meet, or exceed the average?
x = 231,000 divded by 227 boats
x = 1017

Answer: EXCEED

How many days must we average this catch rate to break even?

Answer:
45,000 lbs. needed to break even
x = 45,000 lbs. divided by 1238 x 6.3 lbs.
x = 45,000 lbs. divided by 7,799.4 lbs.

x = 5.77 days to break even (rounded to 6 days)
x = 6 days

Activity 4

Mike has a load of fish to drop off at the tender, which is three miles away. Mike is traveling at a speed of eight knots. How long will it take Mike to reach the anchored tender? Time = Distance divided by Rate ($T = D/R$)

T = 3/8
T = .375 of an hour (60 minutes)
T = .375(60)

Answer: 22.5 minutes

On another day Mike is traveling to a tender 5 miles away. The tide is in full flood and Mike is bucking a 3 knot current. Mike has the boat running at a water speed of 10 knots. How long will it take for Mike to reach the tender? Round the answer to the nearest minute.

Mike's actual speed is 7 knots (10-3)
T=5/7
T = .714 (60 minutes)

Answer: 43 minutes

Now let's say that the tide is in full ebb and Mike and Dave are heading toward an anchored tender. Mike is traveling with the tide, which is running at 4 knots. He is traveling at 9.5 knots speed over the water. Dave is headed against the tide and is traveling at 12 knots over the water. If Mike is 7 miles from the tender and Dave is 4 miles from the tender, who will get there first and by how many minutes? (*Round to the nearest minute.*)

Mike
T = 7 / 13.5 (Mike's Actual Speed)
T = .518 of an hour
T = 31 minutes

Dave
T = 4/8
T = .5 of an hour
T = 30 minutes

Answer: Dave will arrive at the tender first by one minute.

Activity 5

After delivering the catch to the tender, we fuel up. Reading the Hobbs meter at fill up, we learn that we have used 245 gallons of diesel during three days of fishing. At the previous fill up, the boat took on 223 gallons of diesel after two days of fishing. Given the total gallons of fuel, what is our average fuel consumption rate per day?

245 + 233 = 468 gallons of diesel fuel

468/5 = 93.6 gallons of diesel fuel

Answer: 93.6 gallons of diesel fuel

If we burn this amount of fuel for the days needed to break even, and the cost is $4.39 per gallon, will this agree with our fuel cost estimate? (*In other words, is our estimated cost of fuel to break even too low, right on, or too high?*)

Number of days to the breakeven point = 6

$4.39 x 93.6 x 6 = $2,448.58

Answer: $2,448.58 -- the estimate was too high.

Activity 6

By the end of the season, we caught 94,650 pounds of sockeye salmon at a fixed price of $1.40 per pound. We also incurred the following expenses: (*See Boat Payment and Insurance in Exercise 2.*)

Crew Pay	25% of gross (i.e. total revenue)
Travel	$1,745
Diesel fuel	$5,986
Maintenance	$5,750
Food	$1,825
Miscellaneous	$1,645

How much profit did we make after expenses?

94,650 pounds x $173 = $166,584 (total revenue)
Crew Share = $41,646

$41,616 + $9,000 + $2,300 + $1,745 + $5,986 + $5,750 + $1,825 + $3,758 = $94,604

Profit after expenses is $166,584 - $94,604 = $71,980

Answer: $71,980

Algebra Lesson 5

Tlingit Phrase: Kei k̲ugux̲sa.áat´.

English Translation: It will be cold.

Lesson 5 - Glacier Calculations

Southeast Alaska is a land of glaciers. Our maritime climate and coastal mountains create the conditions which favor the growth and ice fields and glaciers. There are over 100,000 glaciers in Alaska. Over half of the earth's mountain glaciers are found in Alaska. Glaciers cover five percent of the state.

Glaciers are dynamic rivers of ice which have sculptured the mountains and valleys of Southeast for thousands of years. As a glacier moves forward, or advances, it is also melting. If the melt rate exceeds the rate of advancement, we say the glacier is receding. Today, most of the glaciers in Southeast are receding. One of the most dramatic examples of a receding glacier is located in the Mendenhall Valley near Juneau.

The Juneau Ice field, which receives over 100 feet of snow each year, is the birthplace of 38 glaciers. The most notable of these is the Mendenhall Glacier. As America's most visited glacier, the Mendenhall Glacier attracts over 500,000 visitors each year. The Mendenhall Glacier is truly magnificent. At over 12 miles long and over a half mile wide at the terminus (*face*), this river of ice is an awesome sight.

Let's examine the dynamic changes in the Mendenhall Glacier using mathematics.

Activity 1

CALCULATING THE RATE OF ICE LOSS

The Mendenhall glacier is receding. This doesn't mean that the glacier is moving backwards. It means that pieces (*icebergs*) are breaking and falling into Mendenhall Lake faster than the glacier is moving forward. Given the rate at which it is receding, we will calculate how fast ice is breaking off the glacier.

In order to calculate the rate ice is breaking off the glacier we need to subtract the total loss of ice from the flow rate. The first step is finding the flow rate from the formula:

Flow rate = distance/time

After measuring the glacier's movement over the period of one day we find it has moved 6 inches, which is .5 feet. When we express this in scientific notation it is:

$$Flow\ rate = \frac{5*10^{-1}\ ft}{1*10^{0}\ day} = \frac{5*10^{-1}\ ft}{1\ day} = 5*10^{-1}\ ft/day$$

Notice that the zero exponent, 10^0, is equal to 1. This is because the exponent of a number tells us to multiply 1 by the number as many times as the exponent says. For example: = (1) (10) (10) (10) = 1000. However, with zero exponents 1 isn't being multiplied by anything, therefore any number, n,
with a zero exponent, $n^0 = 1$.

We also know that the glacier has lost 382.52 feet of ice over the past year, but we need to find the average ice loss per day.

$$Average\ Ice\ loss\ per\ day = \frac{annual\ ice\ loss}{days\ in\ a\ year}$$

When we express this in scientific notation we have:

Average ice loss per day $= \dfrac{3.8252*10^{2}}{3.65*10^{2}} = \dfrac{3.8252}{3.65} * \dfrac{10^{2}}{10^{2}} = 1.048\ ft/day$

Now we can subtract the total loss of ice from the flow rate:

$$Average\ Ice\ loss\ per\ day = \left(5*10^{-1}\ ft/day\right) - \left(10.48*10^{-1}\ ft/day\right) = -5.48*10^{-1}\ ft/day = -0.548\ ft/day$$

Our result is negative because the average loss of ice per day (1.048 ft / day) is greater than the flow rate of the glacier ($5*10^{-1}$ ft / day). Therefore, the average recession rate of the glacier is -0.548 ft / day. So, even though the glacier moves forward 182.5 ft / year, because it loses 382.52 ft / year of ice we have a recession of -200 ft / year. To check our calculation we multiply the average daily recession rate with the total number of days in the year:

$$\frac{-1.048\ ft}{1\ day} * \frac{365\ days}{1\ year} = \frac{-1.048*365}{1*1} * \frac{ft}{year} * \frac{days}{day} = \frac{-200}{1} * \frac{ft}{year} * 1 = -200\ ft/year$$

Our answers are in agreement, so our calculations are correct.

Now you try it! Using the formulas above, calculate how fast ice is breaking off of a hypothetical glacier.

Glacier A: Ice flow rate is 3 inches a day, and the yearly ice loss is 175 ft.

$$Flow\ rate = \frac{2.5*10^{-1}\ ft}{1*10^{0}\ day} = \frac{2.5}{1} * \frac{10^{-1}}{1} * \frac{ft}{day} = 2.5*10^{-1}\ ft/day$$

$$Average\ ice\ loss\ per\ day = \frac{1.75*10^{2}\ ft}{3.65*10^{2}\ day} = \frac{1.75}{3.65} * \frac{10^{2}}{10^{2}} * \frac{ft}{day} = .479\ ft/day$$

$$Average\ ice\ loss\ per\ day = \left(2.5*10^{-1}\ ft/day\right) = \left(4.79*10^{-1}\ ft/day\right) = -2.29*10^{-1}\ ft/day = -0.229\ ft/day$$

$$\frac{-0.479\ ft}{1\ day} * \frac{365\ days}{1\ year} = \frac{-0.479*365}{1*1} * \frac{ft}{year} * \frac{days}{day} = \frac{-175}{1} * \frac{ft}{year} * 1 = -175\ ft/year$$

Our answers are in agreement (*rounded to the nearest whole digit*), so our calculations are correct.

Activity 2

Given the dimensions of an iceberg visible above the surface, we will calculate an algebraic expression for estimating the total volume. (*Multiplication of Polynomials.*)

The density of pure ice water is 920 kg/m^3 and the density of seawater is 1025 kg/m^3, which is a ratio of approximately 8/9. Therefore, only 1/9th of the total mass of an iceberg is visible above the surface. Given this information, we only need to measure the visible shape of an iceberg to estimate its total volume. To simplify this process we use the volume formula for a pyramid (Area of the Base * Height * 1/3). Remember that the base equals length multiplied by width. (*h = height, B = base*)

$$V_{visible} = 1/3\,hB, \text{ where } V_{visible} = \text{visible volume},\ h = height,\ B = base$$

After substituting the variables $h = x, B = 2xy + 1$ in the formula our equation now is:

$$V_{visible} = 1/3x(2xy+1)$$

Since the total mass of the iceberg is proportional to the total volume, and we know that only 1/9th of the iceberg is visible, we will calculate the total volume by multiplying the visible volume by a factor of 8.

$$V_{total} = 8[1/3\,x(2xy+1)]$$

When we simplify the expression the resulting equation is:

$$V_{total} = 2/3x^2y + 1/3x$$

The dimensions given for our iceberg are: x = 25 ft, y = 30 ft, so our equation yields:

$$V_{total} = 2/3\left[625(30)\right] + 1/3(25) = 12500 + 8.33 = 12508.33 \text{ cu ft}$$

What would the total volume be if x = 50, y = 67?

V_{total} = 2/3[2500(67)] + 1/3(50) = 111666.67 + 16.67 = 111683.33 cu ft

What if we set $h = x^2, B = (3x+1)(y-1)$?

What does our equation for the total volume of an iceberg look like now?

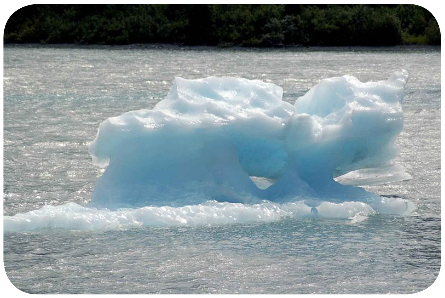
V_{total} = $8\{1/3x^2[(3x+1)(y-1)]\}$

Simplifying the expression yields:

V_{total} = $8x^3y + (-8)x^2 + 2.67x^2y + (-2.67)x^2$

If we set x = 5, y = 7, what value does our formula yield?

V_{total} = 7000 + (-200) + 467.25 + (-66.75) = 7,200.5 cu ft



Activity 3

Given a specific quantity of icebergs, we will calculate an expression for the total volume of ice breaking off over a specific period of time (*adding polynomials*).

Using the formula (V=1/3hB) from **Exercise #3** for the volume of our icebergs, we need to determine how many icebergs have calved from the glacier over a given period of time. After counting and measuring the icebergs that have calved in two days, we find that there are 3 icebergs with varying dimensions:

Iceberg #1: h = x B = (x + y) y

Iceberg #2: h = x B = x (y + 1)

Iceberg #3: h = x B = (x - y) (y - 2)

Inserting the dimension variables for each iceberg generates a polynomial formula for each volume, which we then simplify:

Iceberg #1 = $1/3x[(x+y)y] = 1/3x^2y+1/3xy^2$

Iceberg #2 = $1/3x\ [x(y+1)] = 1/3x^2y+1/3x^2$

Iceberg #3 = $1/3x[(x-y)(y-2)] = 1/3x^2y+(-2/3)x^2+(-1/3)xy^2+2/3xy$

Our equation will take the sum of the simplified polynomials for each iceberg volume multiplied by the period of time for our observations, which is two days:

Volume*Time = $[1/3x^2y+1/3xy^2) + (1/3x^2y+1/3x^2) + (1/3x^2y+-2/3x^2+-1/3xy^2+2/3xy\}]$2days

Simplified, this yields our formula for the total volume of ice lost over a period of two days:

$$V_{total} = 2x^2y+(-0.67)x^2+1.33xy \text{ cu.ft}$$

What value does our formula yield if we set x = 30, y = 40?

V_{total} = 72,000 + (-603) + 1,596 = 72,993 cu ft

Now you try it! Given the dimensions for icebergs #4, #5, and #6, calculate the total volume of ice breaking off over a period of 5 days using the simplified formula .

Iceberg #4: $h=x^2$ B = (x + 1) y

Iceberg #5: $h=x^2$ B = x (y + 2)

Iceberg #6: $h=x^2$ B = x (y - 2)

Inserting the dimension variables for each iceberg generates a *polynomial formula* for each volume, which we then simplify:

Iceberg #1 = $1/3x^2[(x+1)y] = 1/3x^3y + 1/3x^2y$

Iceberg #2 = $1/3x^2[x(y+2)] = 1/3x^3y + 2/3x^3$

Iceberg #3 = $1/3x^2[x(y-2)] = 1/3x^3y + (-2/3)x^3$

Our equation will take the sum of the simplified polynomials for each iceberg volume multiplied by the period of time for our observations, which is 5 days:

Volume*time = $[1/3x^3y + 1/3x^2y) + (1/3x^3y + 2/3x^3) + (1/3x^3y + -2/3x^3)]5$

Simplified, this yields our formula for the total volume of ice lost over a period of 5 days:

$$V_{total} = 5x^3y + 1.67x^2y \text{ cu ft}$$

What value does our formula yield if we set ?

$$V_{total} = 388,125 + 8,642.25 = 396,767.25 \text{ cu ft} = 3.9676725 \times 10^5 \text{ cu ft}$$

Additional Information about the Mendenhall Glacier

The Mendenhall Glacier formed the Mendenhall Lake as the glacier receded in the early part of the 20th century. As the glacier melts into the lake, the rate of recession has accelerated. According to the scientists who monitor the rates of glacier recession in Alaska, the recession rate of the Mendenhall Glacier will slow down once the glacier has receded above the lake. For those of us who appreciate the mighty Mendenhall Glacier and the half million tourists who visit this site annually, this is good news.

Geometry Lesson 1

Tlingit Phrase: Ligéi.

English Translation: It is tall.

Lesson 1 - How High is it?

We live in one of the most dramatic landscapes on earth. Thousands of visitors come to Southeast Alaska each year and marvel the majestic landscape. The ice field above Juneau has been described by visitors as a cathedral of rock and ice. We, who live here year-round, are accustomed to seeing tall trees, glaciers, and mountains. As we move from one place to another, whether on foot, by car, boat or small airplane, we constantly orient ourselves by monitoring the visual cues from the landscape.

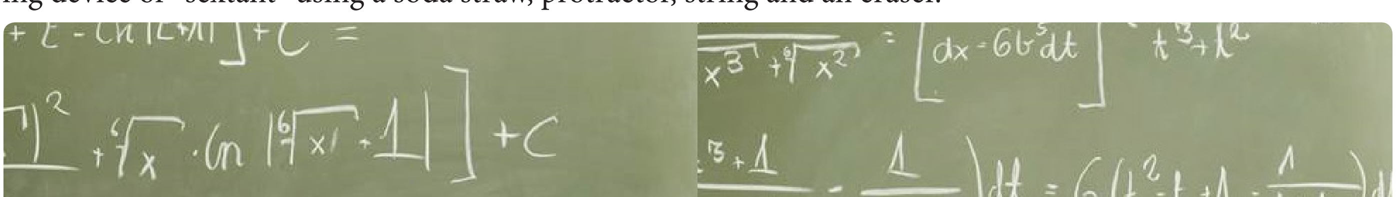
The communities in Southeast are located along fjords and in bays surrounded by mountains. Flat land is a limited resource in Southeast. An Alaskan who is abruptly thrust into totally flat environment may experience a sense of disorientation and confusion. The seemingly unchanging typography may appear boring and featureless to one who is accustomed to the dramatic variations provided by mountains and glaciers. A Southeasterner may experience difficulty determining his or her location and finding direction in an area where there is little change in elevation. A Southeast fisher who fished for several days off the coast near Virginia Beach, Virginia, reported that he finally realized the need for lighthouses on the East Coast. The coastline is virtually flat and featureless from a mile to two offshore. The only discernible features to the mariner were the lighthouses.

Have you ever wondered how tall a large tree actually is, or how tall that mountain across the bay is, or how tall a building is? One way of determining height is to simply measure the height with a tape measure. This is simple to do with small object such a post or single story building. But try to directly measure the height of tall tree, eight story building, or a mountain! Direct measurement may be impossible. That is where math becomes very useful. We can measure the height of tall objects quite accurately by indirectly by using mathematics. Here is how we can do it.

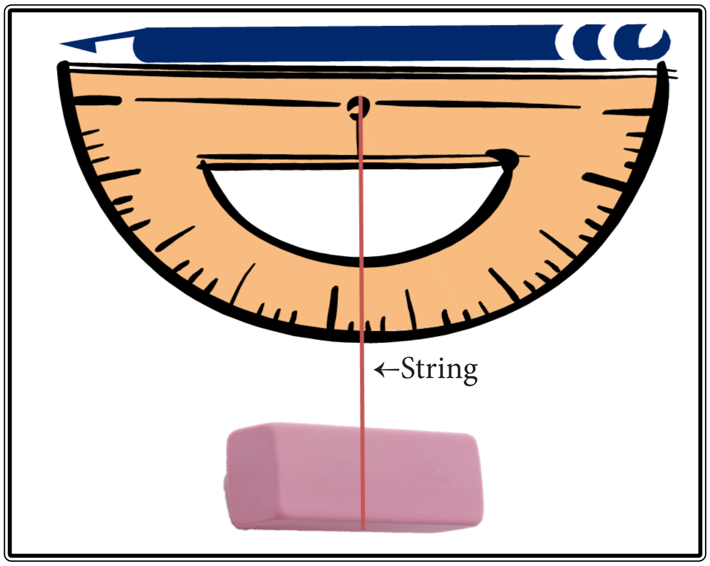
Historical Math Fact: Indian mathematician Radhanath Sikdar was the first person to identify Mount Everest as the tallest mountain on earth. Using the method we are about to learn, he accurately measured the altitude of Mount Everest from India, 150 miles away from the mountain. Early calculations of the height of the mountain led to an interesting conclusion. In the early 1950's a British expedition went to Nepal to determine the exact altitude of Mount Everest. The altitude was declared to be exactly 29,002 feet above sea level. When the expedition returned to Britain, the scientists admitted that the actual calculations revealed that Mount Everest was exactly 29,000 feet tall. The mathematicians arbitrarily added two feet to the official altitude of Mount Everest to avoid giving the impression that the height determined by their expedition was nothing more than an estimate.

Height (*altitude*) calculations are actually easy to make. **First, you will need to determine the baseline–that is the distance from your position to the base of the object that you are measuring**. For an object such as a tree or a building, simply use a tape or other measuring device and determine the distance. For a mountain, you can determine your location and your distance from the center of the base of the mountain using a map.

The second step is to measure the angle of from where you are standing to the top of the object you are measuring. You will need a measuring device to measure this angle. You can make a simple angle measuring device or "sextant" using a soda straw, protractor, string and an eraser.



Activity 1



The angle is found by subtracting the reading from 90 degrees. With your "sextant," determine the angle to several tall landscape features (*such as mountains, tall trees, buildings, flagpole, etc.*) which you can clearly see and measure. Remember: determine the angle by subtracting your reading from 90 degrees. Suggestion – do this in pairs. One person sights the sextant and the other person reads the angle. Record your reading below:

Feature Being Measured	**Angle**
________________________	__________
________________________	__________
________________________	__________
________________________	__________
________________________	__________

The third step is to consult a Tangent Table and find the Tangent of the angle.

Angle	tan(a)
0.0	0.00
1.0	.0175
2.0	.0349
3.0	.0524
4.0	.0699
5.0	.0875
6.0	.1051
7.0	.1228
8.0	.1405
9.0	.1584
10.0	.1763
11.0	.1944
12.0	.2126
13.0	.2309
14.0	.2493
15.0	.2679
16.0	.2867
17.0	.3057
18.0	.3249
19.0	.3443
20.0	.3640
21.0	.3839
22.0	.4040
23.0	.4245
24.0	.4452

Angle	tan(a)
25.0	.4663
26.0	.4877
27.0	.5095
28.0	.5317
29.0	.5543
30.0	.5773
31.0	.6009
32.0	.6249
33.0	.6494
34.0	.6745
35.0	.7002
36.0	.7265
37.0	.7535
38.0	.7813
39.0	.8098
40.0	.8391
41.0	.8693
42.0	.9004
43.0	.9325
44.0	.9657
45.0	1.000

Angle	tan(a)
46.0	1.0355
47.0	1.0724
48.0	1.1106
49.0	1.1504
50.0	1.1918
51.0	1.2349
52.0	1.2799
53.0	1.3270
54.0	1.3764
55.0	1.4281
56.0	1.4826
57.0	1.5399
58.0	1.6003
59.0	1.6643
60.0	1.7321
61.0	1.8040
62.0	1.8907
63.0	1.9626
64.0	2.0503
65.0	2.1445
66.0	2.5460
67.0	2.3559
68.0	2.4751
69.0	2.6051
70.0	2.7475

Angle	tan(a)
71.0	2.9042
72.0	3.0777
73.0	3.2709
74.0	3.4874
75.0	3.7321
76.0	4.0108
77.0	4.3315
78.0	4.7046
79.0	5.1446
80.0	5.6713
81.0	6.3138
82.0	7.1154
83.0	8.1443
84.0	9.5144
85.0	11.430
86.0	14.301
87.0	19.081
88.0	28.636
89.0	57.290
90.0	INFINITE

The fourth and final step is to determine the height by multiplying baseline by the tangent of the angle. The product is the height of the object you are measuring. For example, you are 10 meters away from a flagpole and you have determined the angle is 45 degrees. The tangent of 45 degrees is 1.0. Multiplying 10 by 1.0 you determine that the height of the flagpole is 10 meters.

Why does this work? What exactly is the tangent? A tangent is a function. In math, a function is a relationship between two numbers. For example, let us say that you are buying several cans of soup at the local store. Each can, X, costs $1.85. Let us assume that the variable Y is the total cost you will pay for the number of cans you buy. The function, or relationship between these two variables, can be expressed as:

Y = 1.85X

You bought one can of soup. Then Y, the total cost, is $1.85. If you bought two cans of soup, Y = $2.70. This function can be expressed as follows: **The tangent of X is 1.85**
Let's use this function to determine Y (*the total cost*) of several different values of X (*the total number of cans*).

If X = 3, then Y = $5.55
If X = 4, then Y = $7.40
If X = 5, then Y = $9.25

What would be the value of Y if X = 45? $83.25

In math shorthand, this function is written as **tan(X) = 1.85**

The tangent of the angle is a function defined as: $\mathbf{\tan x = \frac{\sin x}{\text{cosine } x}}$

Activity 2

With a partner, determine the height of a nearby building.

Follow these four steps:

With a tape measure or other measuring device, measure the distance between yourself and the building.

____
(Distance)

Determine the angle of sight to the top of the building. (Remember to subtract your reading from 90 degrees when using your homemade sextant.)

____
(Angle)

Look up the tangent of the angle in the tangent table.

____
(Tangent of the Angle)

Now, multiply the baseline (your distance from the building) by the tangent of the angle.

____
(Height of the Building)

Activity 3

With a partner, use this method to determine the height of four manmade or natural features. If you are measuring the height of a mountain which is a significant distance from you, use a map or some other resource to determine the distance.

1) ______________________________ ______________________________

(Object or Geographic Feature) (Distance)

______________________________ ______________________________

(Angle) (Tangent of the Angle)

____

(Height)

2) ______________________________ ______________________________

(Object or Geographic Feature) (Distance)

______________________________ ______________________________

(Angle) (Tangent of the Angle)

____

(Height)

3) ______________________________ ______________________________

(Object or Geographic Feature) (Distance)

______________________________ ______________________________

(Angle) (Tangent of the Angle)

____

(Height)

4) ______________________________ ______________________________

(Object or Geographic Feature) (Distance)

______________________________ ______________________________

(Angle) (Tangent of the Angle)

____

(Height)

Geometry Lesson 1 Extension

To better understand *sine*, *cosine* and *tangent*, let's take a look at these triangles. Begin with the definitions. We start with a right triangle. The side opposite from the right angle is the hypotenuse, "**h**". This is the longest side of the three sides of a right triangle

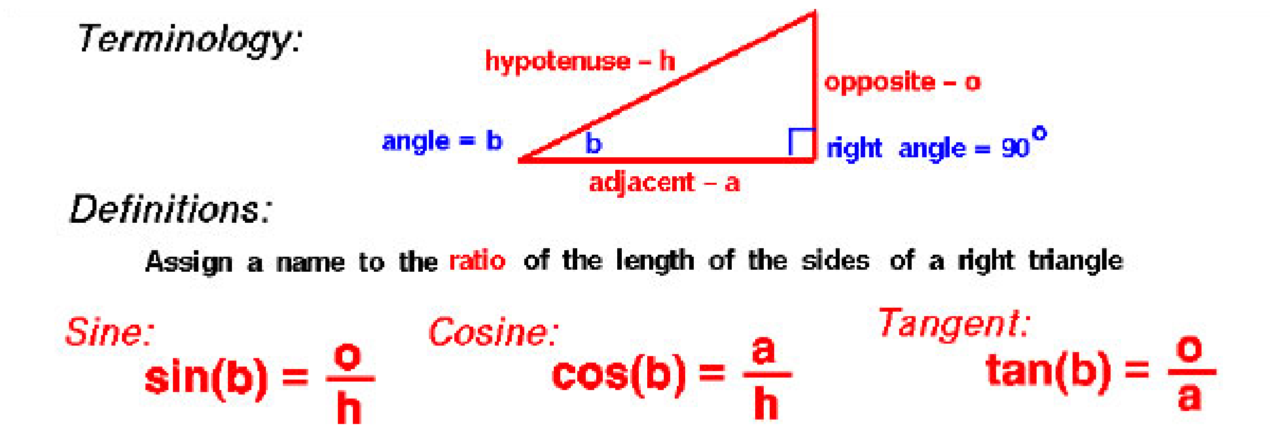


Photo Credit: *NASA, Glenn Research Center*

We pick one of the other two angles and label it **angle b**. Since the sum of all angles of a triangle is 180 degrees, if we know the value of **b**, then we know that the value of the third angle is 90 - **b**. (Remember–a right triangle has one angle that is 90 degrees.) The side opposite, the **angle b**, we will call **o** for "*opposite*". The remaining side we will label a for "*adjacent*". The three sides of the triangle are labeled **o**, **a** and **h**. The sides **a** and **h** make up **angle b**.

The ratio of the right sides of a right triangle depends only on the value of the **angle b**.
We define the ratio of the opposite side of the hypotenuse to the *sine* of the **angle b** and give it the symbol **sin(b)**.

$$\sin(b) = o/h$$

The ratio of the adjacent side to the hypotenuse is called the *cosine* of the **angle b** and given the symbol **cos(b)**. It is called the *cosine* because its value is the same as the *sine* of the other angle in the triangle which is not the right angle.

$$\cos(b) = a/h$$

Finally, the ratio of the opposite side to the adjacent side is called the tangent of the angle b and is given the symbol tan(b).

$$\tan(b) = o/a$$

To demonstrate the value of *sine*, *cosine* and *tangent*, let's look at these three triangles on page 46.

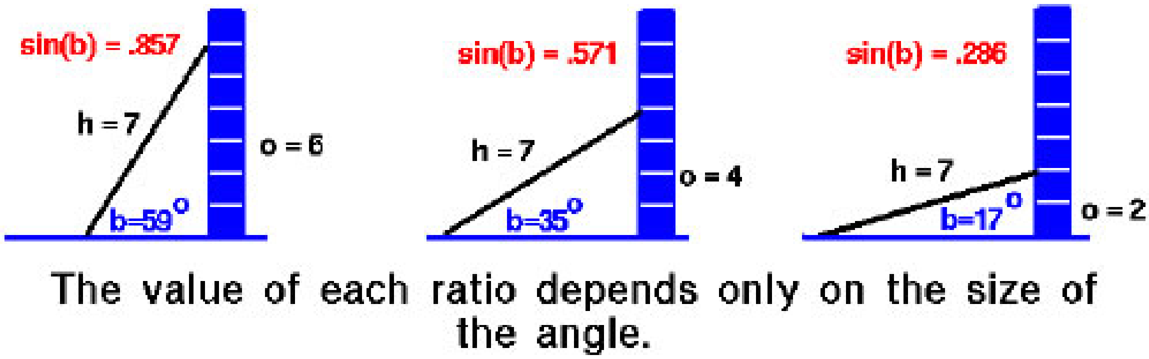


Photo Credit: *NASA, Glenn Research Center*

In the first example, we have a 7 foot ladder that we lean against a wall. The wall is 7 feet high. We have drawn white lines on the wall at one foot intervals. The length of the ladder is fixed. If we incline the ladder so that it touches the 6 foot line, the ladder forms an angle of nearly 59 degrees to the ground. The ladder, ground and wall form a right triangle. The ratio of the height on the wall (*o - opposite*) to the length of the ladder (*h - hypotenuse*) is 6/7, which equals roughly .857. This ratio is defined to be the sine of b = 59 degrees. The ratio stays the same for any triangle with a 59 degree angle.

In the second example, we incline the 7 foot ladder so that it only reaches the 4 foot line. As shown in the figure, the ladder is no inclined at a lower angle than in the first example. The angle is about 35 degrees. The ratio, of the opposite of the hypotenuse, is now 4/7, which equals roughly .571.

In the third example, the 7 foot ladder only reaches the 2 foot line. The angle decreases to about 17 degrees and the ratio is 2/7, which is about .286. As you can see, for every angle there is a unique point on the wall that the 7 foot ladder touches. It is the same point every time we set the ladder to that angle. Mathematicians call this situation a **function**.

Since the *sine*, *cosine* and *tangent* are all functions of the **angle b**, We can measure the ratios once and produce tables of the values of the *sine*, *cosine* and *tangent* for the various values of **b**. Later, if we know the value of an angle in a right triangle, the tables tell us the ratio of the sides of the triangle. If we know the length of any one side, we can solve for the length of the other sides. Or if we know the ratio of any two sides of a right triangle, we can find the value of the angle between the sides.

<u>We can use these tables to solve problems in the real world.</u>

Gunalchéesh!

Goldbelt Heritage Foundation

"Drink from the Vessel of Traditional Knowledge."
~Kashudoha~

This curriculum was brought to you by **Goldbelt Heritage Foundation**, a non-profit organization that was formed in 2007 to document the Tlingit language and stories to preserve our culture and history for future generations. The Foundation seeks to translate the Tlingit oral language into a written language. Due to the complexity of the Tlingit language, the process involves years of effort and research with Tlingit elders and language specialists.